CHARACTER OF SINGULARITY AND STOCHASTIC PHENOMENA IN SELF FOCUSING

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1. When light propagates in a self-focusing medium, local regions of large amplitude - foci - are produced. The present paper is devoted to a study of the structure of the wave field near foci of this type, and also to the structure of a beam with a large number of foci.

From the quasioptical equation [1, 2]

$$i\frac{\partial E}{\partial z} + \frac{1}{2}\Delta_\perp E + |E|^2 E = 0 \tag{1}$$

there follows (see [3]) the relation

$$\int r^2 |E|^2 \, d\mathbf{r} = \frac{1}{2} I_2 z^2 + C_1 z + C_2 . \tag{2}$$

Here

$$I_2 = \frac{1}{2}\int (|\nabla_\perp E|^2 - |E|^4)\, d\mathbf{r}$$

is the integral of (1), and C_1 and C_2 are constants.

When $I_2 < 0$, relation (2) cannot be satisfied for all $z > 0$, since its right-hand side becomes negative as $z \to \infty$. Therefore the propagation of a beam with negative I_2 leads to a certain $z = z_0$ to formation of a singularity in the field.

We put $E = A\exp(i\Phi)$; we then have for A and Φ:

$$\begin{aligned} \frac{\partial A^2}{\partial z} &= -\frac{1}{r}\frac{\partial}{\partial r} r A^2 \frac{\partial \Phi}{\partial r} , \\ A\left(\frac{\partial \Phi}{\partial z} + \frac{1}{2}\left(\frac{\partial \Phi}{\partial r}\right)^2\right) &= A^3 + \frac{1}{2r}\frac{\partial}{\partial r} r \frac{\partial A}{\partial r} . \end{aligned} \tag{3}$$

We assume that near the singularity A and Φ takes the form

$$\begin{aligned} &A = R(r/f(z))/f(z) - A_0 + \delta A + \ldots, \\ &\Phi = \int \frac{dz}{f^2(z)} + \frac{1}{2}\frac{f'(z)}{f(z)} r^2 + \ldots, \\ &f(z_0) = 0, \quad \frac{1}{2r}\frac{d}{dr} r \frac{dR}{dr} + R^3 - R = 0. \end{aligned} \tag{4}$$

Formulas (4) satisfy Eqs. (3) accurate to small terms. For the correction δA we have:

$$\frac{1}{2r}\frac{\partial}{dr}r\frac{\partial}{\partial r}\delta A + \frac{3}{f^2}R^2(r/f)\delta A - \frac{1}{f^2}\delta A =$$

$$= -\frac{3}{f}R^2(r/f)A_0 + \frac{1}{2}R(r/f)\frac{f''}{f^2}r^2 . \qquad (5)$$

The operator

$$L\psi = \frac{1}{2r}\frac{\partial}{\partial r}r\frac{\partial\psi}{\partial r} - 3R^2\psi - \psi$$

has, as shown in [4], exactly one finite eigenfunction $\psi_0(r)$. The condition for the solvability of (5) is that its right-hand side be orthogonal to $\psi_0(r/f)$. This condition yields for f

$$f'' = -3\alpha A_0/f^2 , \qquad (6)$$

where

$$\alpha = 2\frac{\int\psi_0(r)R^2(r)r\,dr}{\int\psi_0(r)R(r)r^3dr} .$$

From this we get

$$f \to 3\left(\frac{\alpha A_0}{2}\right)^{1/3}(z - z_0)^{2/3}$$

as $z \to z_0$. Thus, near the singularity the field increases like $A \sim (z_0 - z)^{-2/3}$ and the power concentrated in the singularity is exactly equal to the critical power $I_0 = \int R^2(r)rdr$.

For intense beams ($I = \int A^2rdr >> I_0$) the position of the singularity depends in an unstable manner on the beam shape.

2. Theoretical predictions concerning the character of the singularity were verified by numerically solving Eq. (1). As the initial conditions, we chose Gaussian beams with powers 5 and 13.5 times critical. We succeeded, without appreciable loss of accuracy, in reaching amplitudes on the axis, approximately 100 times larger than the initial values. A reduction of the table of the actual values of A(z) has shown that, accurate to 0.1%, sections of this curve can be approximated by the hyperbolas $A = A_0/(z - z_0)^{\alpha}$, with α/α_0 ranging from ~0.75 at $A \simeq 40$ to 0.9 - 1.1 at $A \simeq 100$. Here $\alpha_0 = 2/3$ is the theoretical prediction.

In accordance with the theory, when $z \to z_0$ the structure of the beam near the axis constitutes a plateau on which there is a sharply pronounced bell-shaped profile. Regardless of the beam power, the power concentrated in the peak was equal to the critical value (accurate to ~5%).

We verified also the dependence of the position of the focus on the initial beam profile. To this end, the initial Gaussian beam shape was perturbed by a sinusoidal increment

$$U(r, 0) = \exp(-r^2/a^2)\left(1 + \epsilon\cos\frac{2\pi r}{3}\right).$$

at $\epsilon \sim 0.1$, the position of the focus was shifted by 50% from the initial position, thus confirming the sensitivity of the position of the focus to the detailed structure of the beam.

3. With the field at the focus limited by multiphoton absorption, the field grows until a power on the order of the critical value is absorbed. If the effective absorption coefficient is $\nu_{eff} = \beta|E|^{2n}$, then the rate of energy absorption is

$$\frac{d}{dz}\int A^2 d\mathbf{r} = r\beta\lambda/f^{2n}(z), \quad \lambda = \int R^{2n+2}(r)\, d\mathbf{r}.$$

From this we have for the field at the maximum

$$E_{max} \sim \left(\frac{1}{\beta}\right)^{2/(4n-3)}.$$

For two-photon absorption (n = 1) we have $E_{max} \sim 1/\beta^2$. If $n < 3/4$, then the nonlinear absorption does not limit the field. In particular, the field is not limited by linear absorption (n = 0).

The field can be limited by saturation of the nonlinearity. This, for example, is the situation with self-focusing in a plasma. Then the nonlinear term in (1) is replaced in the next-higher approximation by $f(|E|^2)E = (|E|^2 - \varepsilon|E|^4)E$. Then Eq. (6) goes over into

$$f'' = -3\alpha A_0/f^2 + \epsilon/f^5. \tag{7}$$

The quantity f now executes periodic oscillations with $f_{min} \sim (\varepsilon/3\alpha A_0)^{1/3}$, corresponding to a periodically oscillating waveguide [5]. Such a waveguide can be treated as a sequence of singularities.

4. The fact that a plane wave is unstable in a self-focusing medium [6] allows us to assume that propagation of an intense beam will be accompanied by development of stochastic phenomena. To verify this, the propagation of beams with $I \sim (10 - 50)I_0$ in a medium with saturation of the nonlinearity and in a medium with three-photon absorption was simulated numerically. In a medium with saturation there occurs, immediately past the first focus, a stochastic picture consisting of a thin filament with power on the order of critical, executing irregular axial oscillations, and a broadly diverging halo, in which stochastic radial and axial oscillations take place. The amplitude of the field in the halo is smaller by 2 - 4 orders of magnitude than in the filament.

In a medium with absorption, at not very large distances from the entry, there is observed the multifocus picture described by Lugovoi and Prokhorov [7, 8]. With increasing z, the character of this picture becomes consecutively more complicated, and at $z \sim 15 - 20$ it becomes utterly stochastic. The largest (~10%) sinusoidal perturbations of the initial beam shift the positions of the foci, decrease their number from 9 or 10 to 5 or 6, and shift the stochasticity boundary to $z \sim 10$.

Thus, the behavior of an intense beam in a self-focusing medium becomes stochastic at any mechanism of amplitude limitation at the focus. The development of stochasticity leads to violation of the radial symmetry and to scattering of the main energy of the beam through an angle $\theta \sim (n_0/n_{nl})^{1/2}$, which for powerful beams greatly exceeds the diffraction angle.

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TRANSPARENCY OF THICK SUPERCONDUCTING FILMS

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In this paper we investigate the possibility of passage of electromagnetic radiation through sufficiently thick films of a pure superconductor; this possibility is connected with the large mean free path of the normal excitations. The field, of frequency $\omega \ll \Delta$, is screened by the Meissner current, mainly at the penetration depth δ. Normal excitations with energy $\varepsilon(p) = (\xi^2 + \Delta^2)^{1/2}$ are accelerated by the electric field: $(\partial p/\partial v)\dot{v} = eE$, where at $\varepsilon - \Delta \ll \Delta$ we have

$$v = v_F \xi/\epsilon(p) \text{ and } \partial v_i/\partial p_k = v_{Fi} v_{Fk}/\Delta. \tag{1}$$

During the time $t \sim \delta/v_z$ that they stay in the skin layer, the excitations acquire a drift velocity v along the electric field

$$\bar{v}(\epsilon) \sim v_F^2 eE\delta/\Delta v_z = eEv_F\delta/\sqrt{\epsilon^2 - \Delta^2}\cos\theta .$$

The current produced by the normal carrier outside the skin layer

$$j_n = e\int \bar{v}(\epsilon)\rho(\epsilon)n(\epsilon)\frac{d\epsilon d\Omega}{4\pi} = \frac{\delta e^2 m p_0 E v_F}{(2\pi)^3}\int_\Delta^\infty \frac{d\Omega}{\cos\theta}\,\frac{\epsilon\, d\epsilon}{\epsilon^2 - \Delta^2}n(\epsilon)$$

will be offset by the current of the "superconducting" electrons $j_s = (-N_s e^2/mc)A_1$, so that the total current in the interior of the plate vanishes. We see therefore that the electric field $E_1 = -i(\omega/c)A_1$, which is connected with the potential A_1, could attenuate in a superconductor more slowly than the total current:

$$E_1 \sim \frac{\omega\delta}{v_F}\,\frac{N}{N_s}E\int\frac{d\mu}{\mu}\,\frac{\epsilon\, d\epsilon}{\epsilon^2 - \Delta^2}n(\epsilon) . \tag{2}$$

The doubly-logarithmic singularity in the integral (2) ($\mu = \cos\theta$) is connected with the singularity in the expression for the excitation velocity (1) along the surface and with the well-known square-root singularity in the state density of the superconductor $\rho(\varepsilon)$. The principal logarithmic terms are determined by the condition that the time required for the excitation to cover the distance $z \gg \delta$ from the surface be small compared with the period of the field: $z/v_z \ll 1/\omega$, with $\varepsilon - \Delta \ll \Delta$. Expressing the electric field at the surface E in terms of the field of the incident wave H_0, namely $E \sim (\omega/c)\delta H_0$,