[3] V.N. Vlasov, V.A. Petrishchev, and V.I. Talanov, Paper at Fourth All-union Conference on Nonlinear Optics, Kishinev, 1970.
[4] V.E. Zakharov, Zh. Eksp. Teor. Fiz. 53, 1735 (1967) [Sov. Phys.-JETP 26, 994 (1968)].
[5] V.E. Zakharov, V.V. Sobolev, and V.S. Synakh, ibid. 60, 136 (1971) [33, 77, 1971)].
[6] V.I. Bespalov and V.I. Talanov, ZhETF Pis. Red. 3, 471 (1966) [JETP Lett. 3, 307 (1966)].
[7] A.L. Dyzhko, V.N. Lugovoi and A.M. Prokhorov, ibid. 6, 655 (1967) [6, 146 (1967)].
[8] V.N. Lugovoi and A.M. Prokhorov, ibid. 7, 153 (1968) [7, 117 (1968)].

TRANSPARENCY OF THICK SUPERCONDUCTING FILMS

L.P. Gor'kov and M.A. Fedorov
L.D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
Submitted 19 October 1971
ZhETF Pis. Red. 14, No. 10, 568 - 572 (20 November 1971)

In this paper we investigate the possibility of passage of electromagnetic radiation through sufficiently thick films of a pure superconductor; this possibility is connected with the large mean free path of the normal excitations. The field, of frequency $\omega \ll \Delta$, is screened by the Meissner current, mainly at the penetration depth δ. Normal excitations with energy $\varepsilon(p) = (\xi^2 + \Delta^2)^{1/2}$ are accelerated by the electric field: $(\partial p/\partial v)\dot{v} = eE$, where at $\varepsilon - \Delta \ll \Delta$ we have

$$v = v_F \xi/\epsilon(p) \text{ and } \partial v_i/\partial p_k = v_{Fi} v_{Fk}/\Delta. \tag{1}$$

During the time $t \sim \delta/v_z$ that they stay in the skin layer, the excitations acquire a drift velocity v along the electric field

$$\tilde{v}(\epsilon) \sim v_F^2 eE\delta/\Delta v_z = eEv_F\delta/\sqrt{\epsilon^2 - \Delta^2}\cos\theta.$$

The current produced by the normal carrier outside the skin layer

$$j_n = e\int \tilde{v}(\epsilon)\rho(\epsilon)n(\epsilon)\frac{d\epsilon\, d\Omega}{4\pi} = \frac{\delta e^2 m p_0 E v_F}{(2\pi)^3}\int_\Delta^\infty \frac{d\Omega}{\cos\theta}\,\frac{\epsilon\, d\epsilon}{\epsilon^2 - \Delta^2}n(\epsilon)$$

will be offset by the current of the "superconducting" electrons $j_s = (-N_s e^2/mc)A_1$, so that the total current in the interior of the plate vanishes. We see therefore that the electric field $E_1 = -i(\omega/c)A_1$, which is connected with the potential A_1, could attenuate in a superconductor more slowly than the total current:

$$E_1 \sim \frac{\omega\delta}{v_F}\frac{N}{N_s}E\int\frac{d\mu}{\mu}\;\frac{\epsilon\, d\epsilon}{\epsilon^2 - \Delta^2}n(\epsilon). \tag{2}$$

The doubly-logarithmic singularity in the integral (2) ($\mu = \cos\theta$) is connected with the singularity in the expression for the excitation velocity (1) along the surface and with the well-known square-root singularity in the state density of the superconductor $\rho(\varepsilon)$. The principal logarithmic terms are determined by the condition that the time required for the excitation to cover the distance $z \gg \delta$ from the surface be small compared with the period of the field: $z/v_z \ll 1/\omega$, with $\varepsilon - \Delta \ll \Delta$. Expressing the electric field at the surface E in terms of the field of the incident wave H_0, namely $E \sim (\omega/c)\delta H_0$, we obtain for the transmission coefficient at $\Delta \sim T$ ($N \sim N_s$):

$$\left|\frac{E_1}{H_0}\right|^2 \sim \left|\frac{\omega^2\delta^2}{v_F c}\ln^2\frac{\omega z}{v_F}\right|^2 .$$

The foregoing considerations are confirmed by an exact calculation. Let us consider first the incidence of an electromagnetic wave on a bulky superconductor (half-space), assuming that the reflection of the electrons from the boundary is specular. In this case the solution takes the form (see, e.g., [1])

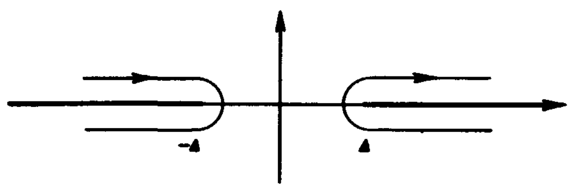
$$A(z) = \frac{H_0}{\pi}\int_{-\infty}^{+\infty}\frac{dq\, e^{iqz}}{q^2 + K_0(q) + K_1(q\omega)} . \qquad (3)$$

Here the function $K = K_0 + K_1$ is the kernel that relates the current with the field in the linear theory (K_0 is the kernel of the static problem [2]).

In the calculation of K_1 it is convenient to start from the expression for the total kernel K, written in the temperature technique, and then carry out an analytic continuation along the real-frequency axis [3]. Under the condition $\omega/T \ll 1$ we obtain

$$K_1 = -\frac{3\pi Ne^2}{8mc^2}\frac{\omega}{T}\int_c \frac{d\epsilon}{\mathrm{ch}^2\epsilon/2T}\,\frac{\epsilon\epsilon_1 + \Delta^2 - \tilde{\gamma}\gamma_1}{\tilde{\gamma}\gamma_1}\int_{-1}^{+1}\frac{(1-\mu^2)\,d\mu}{\tilde{\gamma} + \gamma_1 + qv\mu} .$$

In this formula $\varepsilon_1 = \varepsilon - \omega$, $\gamma_1 = \gamma(\varepsilon_1)$, and γ and $\tilde{\gamma}$ are the roots of $(\varepsilon^2 - \Delta^2)^{1/2}$, determined with the cut $(-\infty, -\Delta)$ (δ, ∞) and taken, respectively, approaching the real axis in the complex ε plane from below and from above, respectively. A unique choice of the branch is determined by the condition $\gamma(0) = i\Delta$, at which $\mathrm{Im}\,\gamma \geq 0$ in the entire plane. The integration is along a contour enclosing the cut (see the figure).



The behavior of the field A(z) at $z \gg v/\Delta$ (in the London case, $z \gg \delta_L$) is determined by the properties of the kernel K at small momenta $qv \ll \Delta$. As to K_0, in this momentum region we have

$$K_0 = 4\pi N_s e^2/mc^2 \equiv \delta_L^{-2} ,$$

where N_s is the density of the "superconducting" electrons, and δ_L is the London depth of penetration. With respect to K_1 it can be shown that at complex qv this function has branch points $qv = \pm\omega$ and $qv = \pm\sqrt{2\Delta\omega}$, and therefore the characteristic scales over which it varies are the values $qv \sim \omega$ and $qv \sim \sqrt{2\Delta\omega}$.

For $qv \ll \omega$, we obtain after simple calculations

$$K_1(0) = 4\pi(N - N_s)e^2/mc^2 .$$

We note that in this case $K = K_0 + K_1 = 4\pi Ne^2/mc^2$, as should be the case in accordance with Galilean invariance.

In the region $qv \gg \omega$, the function K_1 decreases with increasing q. This enables us to calculate the integral (3) by expanding the integrand with respect to K_1. Satisfaction of the inequality $K_0 \gg K_1$ is equivalent in final analysis to an upper bound in z on the distances considered. At $N \sim N_s$, i.e., at intermediate temperatures, the results obtained above are valid for $z \ll v/\omega$.

Thus, writing out (3) for small K_1 and neglecting the term q^2 compared with K_0 (since $z \gg v/\Delta$), we get

$$A(z) = A_0(z) + A_1(z) = A_0(z) - (\pi K_0(0))^{-1} \bar{A}_0 \int_{-\infty}^{+\infty} dq K_1(q) e^{iqz} \quad (4)$$

(here

$$\bar{A}_0 = \int_0^{\infty} A_0(x)\, dx = H_0 K_0^{-1}(0)).$$

Integrating in (4) with respect to q, ε, and μ, we obtain (assuming that $\omega \ll \Delta^2/T$ and $z \ll v/\omega$) (C is the Euler constant)

$$A_1(z) = i\frac{3}{4}\bar{A}_0 \frac{N}{N_s} \frac{\omega}{v} \frac{\Delta}{T} \operatorname{ch}^{-2} \frac{\Delta}{2T}\, I\left(\frac{\omega}{v} z\right),$$

$$I(\tilde{z}) = \frac{1}{2} \ln^2 \tilde{z} + \left(C + 1/2 - i\frac{\pi}{2}\right)\left(\ln \tilde{z} + \frac{1}{2}\right) + C^2/2 - \pi^2/24 -$$

$$- iC\pi/2 + \left(\ln \tilde{z} + C + \frac{1}{2} - i\frac{\pi}{2}\right)\Phi_1(\Delta/2T) + \Phi_2(\Delta/2T) \quad (5)$$

$$\Phi_1(\beta) = \int_1^{\infty} \frac{dt}{t} f(t), \quad \Phi_2(\beta) = \int_1^{\infty} \frac{dt}{t} f(t) \ln t,$$

$$f(t) = 1 - \frac{t^2}{(t^2-1)^{3/2}} \frac{\operatorname{ch}^2 \beta}{\operatorname{ch}^2 \dfrac{\beta t}{\sqrt{t^2-1}}}.$$

Formulas (5) show that an alternating field is capable of penetrating into a superconductor to a sufficiently large distance $z \lesssim v/\omega$[1]. It is assumed here, of course, that $\ell \gg v/\omega$, where ℓ is the electron mean free path.

Writing down Maxwell's equation in coordinate space, we can readily verify that the expansion in the ratio of K_1 to K_0, and also the possibility of neglecting q^2, correspond exactly to the condition that the total current vanish at large distances $j_s + j_n = 0$, where $j_n = \int K_1(x - x')A_0(x')dx' \approx K_1(x)\bar{A}_0$, in accordance with the considerations advanced in the beginning of the article. It follows therefore that Eqs. (5) retain their form if a diffuse reflection of the electrons from the boundary of the metal is assumed. In this case, however, $A_0 = 3\sqrt{3}H_0\delta^2/4$, where δ is the depth of penetration of the static field in a Pippard semiconductor.

Calculation of the coefficient of transmission of radiation through a relatively thick film is best carried out under the condition of diffuse reflection of the electrons from the film boundary, for in this case the

---

[1]) The presence of a singularity at $qv = \sqrt{2\Delta\omega}$ leads to a dependence of A(z) on the parameter $(\sqrt{2\Delta\omega}/v)z$, but the corresponding terms are small when $\omega \ll \Delta$.

reflected wave can be neglected. As a result we readily obtain

$$\eta(d) = \left|\frac{E_1(d)}{H_0}\right|^2 = \left(\frac{9\sqrt{3}}{16}\,\frac{\omega^2}{cv}\,\delta^2\frac{N}{N_s}\,\frac{\Delta}{T}\right)^2 \mathrm{ch}^{-2}\frac{\Delta}{2T}\left|I(d)\right|^2. \qquad (6)$$

We shall use in the estimates the experimental law $\delta \sim \delta_0(1 - t^2)^{1/2}$, and the velocity v is conveniently expressed in terms of the correlation length $\xi_0 = 0.18\hbar v/kT_c$. For an indium film ($\delta = 6.4 \times 10^{-5}$ cm) of thickness $d = 5 \times 10^{-4}$ cm, on which radiation with wavelength $\lambda = 3$ cm is incident, we obtain $\eta \sim 10^{-11}$ (at $\Delta(0)/2T = 0.9$). An effect of this magnitude lends itself to observation[2)].

In conclusion, the authors are grateful to M.Ya. Azbel', G.M. Eliashberg, and A.I. Rusinov for useful discussions.

[1] A.A. Abrikosov and L.A. Fal'kovskii, Zh. Eksp. Teor. Fiz. 40, 262 (1961) [Sov. Phys.-JETP 13, 179 (1961)].
[2] A.A. Abrikosov, L.P. Gor'kov, and I.E. Dzyaloshinskii, Metody kvantovoi teorii polya v statisticheskoi fizike (Quantum Field Theoretical Methods in Statistical Physics), Fizmatgiz, 1962. [Prentice Hall, 1963].
[3] L.P. Gor'kov and G.M. Eliashberg, Zh. Eksp. Teor. Fiz. 54, 612 (1968) [Sov. Phys.-JETP 27, 328 (1968)].
[4] G.E.H. Reuter and E.H. Sondheimer, Proc. Roy. Soc. A195, 336 (1948).

MANIFESTATION OF THE EXCITON MECHANISM IN THE CASE OF GRANULATED SUPERCONDUCTORS

V.L. Ginzburg
P.N. Lebedev Physics Institute, USSR Academy of Sciences
Submitted 26 October 1971
ZhETF Pis. Red. 14, No. 10, 572 - 575 (20 November 1971)

The exciton mechanism of superconductivity could, in our opinion, become manifest most clearly in the case of systems with planar geometry, i.e., for dielectric-metal-dielectric sandwiches and for layered chemical compounds (see [1] and the literature cited therein). There is no doubt, however, that it is quite difficult to attain an appreciable increase of the critical temperature T_c in the aforementioned cases. For sandwiches, the difficulties are connected with the need for making the metallic film extremely thin, even if we disregard the choice of a suitable dielectric [1, 2]. For layered compounds, the main problem is to introduce the necessary dielectric "layers" (e.g., by "intercalation"). The use of large molecules of the pyridine type [3] for such layers is not effective enough [3, 4]. On the other hand, the possibility of producing "layers" of the semiconductor type with the required exciton band has not yet been demonstrated.

In view of the foregoing, in the new experimental investigations [5 - 7], particular interest attaches to granulated superconductors, in which very

---

[2)]An effect connected with the expulsion of the excitations from the skin layer might exist also in a normal metal. The corresponding calculations are made very difficult by the nonlocality of the problem in the normal metal (in view of the absence of a Meissner current). This question was investigated in [4] only for the limiting case $\omega\tau \ll 1$, which is the inverse of the case considered above.