THEORY OF NUCLEUS FORMATION IN PHASE TRANSITIONS IN MAGNETIC FIELDS

I.A. Privorotskii
L.D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
Submitted 2 November 1971
ZhETF Pis. Red 14, No. 11, 613 - 616 (5 December 1971)

This article considers phase transitions in superconductors and in diamagnetic metals (under conditions of the de Haas-van Alphen effect).

Type I superconductors go over in a magnetic field H_c into the normal state. In a field smaller than H_c ($H_{c2} < H < H_c$; in the Ginzburg-Landau theory [1], $H_{c2} = \kappa\sqrt{2}H_c$), the normal phase can exist only as a metastable (supercooled) one. The metastable state can be disrupted, for example, if thermal activation gives rise to a superconducting-phase nucleus[1)] whose dimensions are large enough so that the growth of the nucleus leads to a decrease of the free energy of the body. The probability of formation of such nuclei is proportional to $\exp(-R_{min}/T)$, where T is the temperature, while the energy barrier R_{min} is the work necessary to produce the so-called critical nucleus: its form is such that a given thickness of the nucleus the free energy of the body is minimal, and the thickness corresponds to the maximum of the free energy. We assume here that the dimensions of the nucleus are large compared with the thickness of the boundary between the normal and superconducting phases, which is valid in the case of weak metastability:

$$1 - H/H_c = \gamma << 1 - H_{c2}/H_c .$$

The work R necessary to produce the nucleus can be represented in the form

$$R = -\gamma\frac{H_c^2}{4\pi}V + \Delta_0\frac{H_c^2}{8\pi}S + \int dV'\frac{(\delta\mathbf{H})^2}{8\pi}, \qquad (1)$$

where V and S are the volume and area of the nucleus, $\Delta_0(H_c^2/8\pi)$ is the surface tension, and $\delta\vec{H}$ is the change of the magnetic field in the normal phase, due to the formation of the nucleus.

In the case of weak metastability ($\gamma << 1$) the critical nucleus turns out to be elongated along the magnetic field $\vec{H}$: $\rho_0(0) << \ell$ (see Fig. 1; the shape of the nucleus is described by the function $\rho_0(z)$, the nucleus is symmetrical with respect to the z axis). It can be assumed that the field $\delta\vec{H}$ is produced by fictitious "magnetic charges," concentrated on the surface of the nucleus. The density of these charges at $\gamma << 1$ is equal to $-(1/4\pi)H_c(d\rho_0/dz)$ (this follows from the condition that the component of the magnetic field $\vec{H} + \delta\vec{H}$ normal to the surface be equal to zero). In the region $\rho << \ell$ we have

$$\delta H_z << \delta H_\rho = -\frac{H_c\rho_0 d\rho_0/dz}{\rho}. \qquad (2)$$

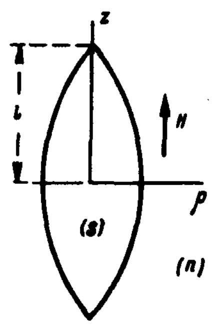

Fig. 1

At $\rho >> \ell$ the field $\delta\vec{H}$ is the dipole field and decreases

1) A normal-phase nucleus can be produced in a superheated superconductor ($H > H_c$) only at the surface of the sample. We do not consider this case.

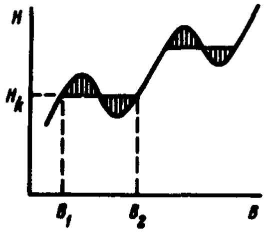

Fig. 2

like $(\rho^2 + z^2)^{-3/2}$. Therefore

$$R = \frac{H_c^2}{4}\int_{-\ell}^{\ell} dz\left\{\ln\frac{\ell}{\rho_0(0)}\rho_0^2\left(\frac{d\rho_0}{dz}\right)^2 + \gamma\rho_0(\rho_m - \rho_0)\right\};\ \rho_m = \frac{\Delta_0}{\gamma}. \quad (3)$$

The saddle point of the functional R, i.e., the critical nucleus, corresponds to a shape given by the equation

$$\frac{z}{\rho_m} = \left(\frac{1}{2\gamma}\ln\frac{1}{\gamma}\right)^{1/2}\left[\arcsin\left(1 - \frac{\rho_0}{\rho_m}\right) + \frac{1}{\rho_m}\sqrt{\rho_0(\rho_m - \rho_0)}\right]. \quad (4)$$

The energy barrier R_{min} is equal to

$$R_{min} = \frac{\pi H_c^2\Delta_0^3}{16\sqrt{2}}\left(\frac{1}{\gamma}\right)^{5/2}\left(\ln\frac{1}{\gamma}\right)^{1/2}. \quad (5)$$

Thermal activation of the nuclei is possible apparently in superconductors with small H_0. No supercooling was observed in such substances [2][2)].

Analogous results are valid also in the case of phase transitions in diamagnetic metals under conditions of the de Haas - van Alphen effect. This phase transition was investigated in [4, 5]. Figure 2 shows the dependence of the field H on the induction B. In the field $H = H_k$, determined from the condition that the shaded areas be equal, coexistence of the phases is possible (the free energies $\tilde{F} = -(1/4\pi)\int_0^H BdH$ coincide). At $H - H_k \neq 0$, one of the phases is metastable. The work R necessary to form the nucleus is in this case equal to

$$R = -|\tilde{F}_2(H) - \tilde{F}_1(H)|V + \frac{\Delta_0(B_2 - B_1)^2}{8\pi}S + \int d^3x\frac{(\delta H)^2}{8\pi}. \quad (6)$$

The field $\delta\vec{H}$ is produced by the "surface charge" $\sigma(z) = \pm[(B_2 - B_1)/4\pi](d\rho_0/dz)$.

The differential permeability of the metastable phase is anisotropic ($\mu = dB_z/dH_z \neq dB_\rho/dH_\rho \simeq 1$). Therefore the logarithmic integral in the expression for the energy of the demagnetization fields is cut off at distances on the order of $\ell/\sqrt{\mu}$.

At small $H - H_K$ we have

$$|\tilde{F}_2(H) - \tilde{F}_1(H)| = \frac{\gamma(B_2 - B_1)^2}{4\pi};\ \gamma = \frac{|H_k - H|}{B_2 - B_1};\ \rho_m = \frac{\Delta_0}{\gamma} \quad (7)$$

(it is assumed that $\gamma \ll 1$, and at $\mu \gg 1$ we have $\gamma \ll \mu^{-1}$). Therefore

[2)] It can be shown that the probability of formation of quantum nuclei of a superconducting phase [3] is exceedingly small, owing to the large mass of the separation boundary. The author is grateful to G.M. Eliashberg for a discussion of this question.

$$R_{min} = \frac{(B_2 - B_1)^2}{4} \int_{-\ell}^{\ell} dz \left[\rho_o^2\left(\frac{d\rho_o}{dz}\right)^2 \ln \frac{\ell}{\rho_o(0)\sqrt{u}} + \gamma\rho_o(\rho_m - \rho_o)\right] =$$

$$= \frac{\pi(B_2 - B_1)^2}{16\sqrt{2}} \Delta_o^3 \frac{1}{\gamma^{5/2}}\left(\ln \frac{1}{\gamma\mu}\right)^{1/2}, \qquad (8)$$

$$\frac{z}{\rho_m} = \frac{\pi}{2}\left(\frac{1}{2\gamma} \ln \frac{1}{\gamma\mu}\right)^{1/2} \left[\arc\sin\left(1 - \frac{\rho_o}{\rho_m}\right) + \frac{1}{\rho_m}\sqrt{\rho_o(\rho_m - \rho_o)}\right]. \qquad (9)$$

The energy barrier R_{min} can change in a very wide range. It is easy to visualize the situation when at T = 1°K the barrier $R_{min} \ll T$. Apparently the thermal activation of the nuclei is possible under the conditions of the experiments of Condon and Walstedt [4]. We note that in this experiment the supercooling was observed only at sufficiently low temperatures.

Similar results for ferromagnets were obtained in a paper by the author [6].

[1] V.L. Ginzburg and L.D. Landau, Zh. Eksp. Teor. Fiz. 20, 1064 (1950).

[2] D. Schonberg, Superconductivity, Cambridge, 1965; E.A. Lynton, Superconductivity, Methuen, 1964.

[3] I.M. Lifshitz and Yu.M. Kagan, Zh. Eksp. Teor. Fiz. 62, No. 1 (1972) [Sov. Phys.-JETP 35, No. 1 (1972)]. S.V. Iordanskii and A.M. Finkel'shtein, ibid. 62, No. 1 (1972) [35, No. 1 (1972)].

[4] J.H. Condon, Phys. Rev. 145, 526 (1966); J.H. Condon and R.E. Walstedt, Phys. Rev. Lett. 21, 612 (1968).

[5] I.A. Privorotskii, Zh. Eksp. Teor. Fiz. 52, 1755 (1967) [Sov. Phys.-JETP 25, 1167 (1967)]. I.A. Privorotskii and M.Ya. Azbel', ibid. 56, 388 (1969) [29, 214 (1969)].

[6] I.A. Privorotskii, ibid. 62, No. 3 (1972) [35, No. 3 (1972)].

"TRANSITION ACCELERATION" OF A CHARGED PARTICLE PASSING THROUGH A BOUNDARY OF AN INVERTED DIELECTRIC

V.P. Gavrilov and A.A. Kolomenskii
P.N. Lebedev Physics Institute, USSR Academy of Sciences

Submitted 3 November 1971
ZhETF Pis. Red. 14, No. 11, 617 - 619 (5 December 1971)

In connection with progress in quantum radiophysics, the question arises of the possibility of transferring energy from inverted media directly to charged particles moving in them. The acceleration of the particle in such media as a result of Cerenkov radiation was considered in [1]. In the present paper we wish to call attention to the possible inversion of the so-called transition radiation [2], which arises when a particle passes through the boundary between vacuum and a dielectric. As will be shown below, on crossing the boundary between vacuum and an inverted dielectric, the ordinary radiation losses can give way to a new effect, wherein energy is transferred to the particle; this effect can be called "transition acceleration."