RESONANT PHENOMENA IN THE EXCITATION OF INTERNAL-STRESS WAVES

V. L. Indenbom and E. M. Shefter
Crystallography Institute, USSR Academy of Sciences
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As is well known, the internal-stress field is described by the equation

$$c_{ijkl}u_{k,jl} - \rho\ddot{u}_i = c_{ijkl}\epsilon^0_{kl,j}\,, \tag{1}$$

where u_i is the displacement vector, c_{ijkl} the elastic-moduli tensor, ρ the density of the material, and ϵ^0_{ij} proper strains of some origin (plastic, temperature, striction, piezo-strains, etc.), which give rise to the elastic strains

$$\epsilon_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}) - \epsilon^0_{ij} \tag{2}$$

and to the corresponding internal stresses

$$\sigma_{ij} = c_{ijkl}\epsilon_{kl}\,. \tag{3}$$

The elastic strains (2) usually turn out to be of the same order of magnitude as the proper strains. Similar results were obtained in many attempts to take into account the inertial forces as applied to different concrete problems (see [1]).

It is seen from (1) that upon suitable choice of even weak sources, the amplitude of the internal-stress waves can be made arbitrarily large. Let us indicate several examples of such a resonant excitation of the internal-stress field, which can be realized, in particular, with modern methods of irradiating bodies with electromagnetic waves.

a) Motion of sources with sonic and supersonic speed. As the speed of the source of internal stresses increases, the elastic field produced by it experiences a "Lorentz" contraction. When the source speed reaches that of sound, resonance sets in and Mach waves begin to be radiated, in analogy with Cerenkov radiation. In the simplest case of an isotropic source ($\epsilon^0_{ij} = \delta_{ij}\epsilon^0$) in an elastic-isotropic medium, the wave equation takes the form

$$\nabla^2\Phi - \frac{1}{c^2}\ddot{\Phi} = \frac{1+\nu}{1-\nu}\epsilon^0. \tag{4}$$

Here Φ is the displacement potential and c the velocity of the longitudinal waves. For a uniformly moving point source, the solution can be written in analogy with the known solution for a uniformly moving electron. The stresses are determined by differentiating the displacement potential.

When $v \gg c$ the phases of the source and the wave coincide; the amplitude of the stresses increases without limit on approaching the generatrix of the radiation cone.

b) Cumulation of waves following instantaneous application of the field. Assume that a proper strain ϵ^0_{ij} is produced in the body instantaneously. Inasmuch as instantaneous displacement of the points of the body is impossible, the stresses at the initial instant are determined by the relation $\sigma_{ij} = -c_{ijkl}\epsilon^0_{kl}$. If ϵ^0_{ij} is constant in the irradiated region, then the initial acceleration of the points on the boundary of this region (the boundary can coincide fully or in part with the free surface of the body) is described by a boundary δ-function.

This results in two waves propagating in opposite sides of the boundary of the irradiated region. If the shape of this region is suitably chosen, cumulative compression of the elastic field can take place, with unlimited growth of the stress amplitude. Of interest in practical problems is the case of uniform excitation of a cylinder of radius R. Solution of (4) yields for the cylinder axis the following stresses:

$$\sigma_{zz} = \sigma_{rr} = \sigma_{\varphi\varphi} = -\frac{\epsilon^0 E}{1-2\nu} \qquad \text{for } t < R/c \tag{5}$$

$$\sigma_{rr} = \sigma_{\varphi\varphi} = -\frac{\epsilon^0 E}{2(1-\nu)}\left[1 + \frac{1}{1-2\nu}\left(1 - ct[c^2t^2 - R^2]^{-\frac{1}{2}}\right)\right] \qquad (t > R/c) \tag{6}$$

$$\sigma_{zz} = -\frac{\epsilon^0 E}{1-\nu}\left[1 + \frac{\nu}{1-2\nu}\left(1 - ct[c^2t^2 - R^2]^{-\frac{1}{2}}\right)\right] \qquad (t > R/c) \tag{7}$$

Here ν is the Poisson coefficient and E is Young's modulus. The amplitude of the relaxation wave becomes infinite at the instant $t = R/c$. As $t \to \infty$ the stress distribution tends to a certain known static solution.

c) Motion of a wave packet. Let a uniformly excited semi-infinite cylinder of radius R move with velocity v along its own axis. Solution of (4) yields for the cylinder axis the following stresses:

for $vt - z < [(v^2/c^2) - 1]^{1/2}R$:

$$\sigma_{zz} = -\frac{\epsilon^0 E}{1-2\nu}\,\frac{1}{1-(c^2/v^2)}, \tag{8}$$

$$\sigma_{rr} = \sigma_{\varphi\varphi} = -\frac{\epsilon^0 E}{2(1-\nu)}\left[1 + \frac{(1-2\nu)^{-1} - (c^2/v^2)}{1-(c^2/v^2)}\right], \tag{9}$$

for $vt - z > [(v^2/c^2) - 1]^{1/2}R$:

$$\sigma_{zz} = -\frac{\epsilon^0 E}{1-\nu}\left\{1 + \left[\left(\frac{1-\nu}{1-2\nu}\right)[1-(c^2/v^2)]^{-1} - 1\right]\left[1 - \frac{vt - z}{\{(vt-z)^2 - [(v^2/c^2) - 1]R^2\}^{1/2}}\right]\right\}, \tag{10}$$

$$\sigma_{rr} = \sigma_{\varphi\varphi} = -\frac{\epsilon^0 E}{2(1-\nu)}\left\{1 + \frac{(1-2\nu)^{-1} - (c^2/v^2)}{1-(c^2/v^2)}\left(1 - \frac{vt - z}{\{(vt-z)^2 - [(v^2/c^2) - 1]R^2\}^{1/2}}\right)\right\}. \tag{11}$$

The cumulation of the relaxation wave occurs in analogy with case (6) and corresponds to crossing of the radiation-cone surface.

At a distance r from the cylinder axis, the maximum stresses are proportional to $\sqrt{R/r}$. An estimate by the method of [2] has shown that the cumulation of the relaxation wave may be the cause of destruction of bodies by laser radiation, where formation of cracks on the axis of a relatively broad beam, which (on the average) does not produce large stresses, is frequently observed [3].

[1] G. Parkus, Nonstationary Temperature Stresses, Fizmatgiz, 1963.

[2] V. L. Indenbom, FTT 3, 2071 (1961), Soviet Phys. Solid State 3, 1506 (1962).
[3] B. M. Ashkinadze, V. I. Vladimirov, et al., JETP 50, 1187 (1966), Soviet Phys. JETP 23 in press (1966).

SPECTRUM OF LIGHT SCATTERED BY DENSITY AND ANISOTROPY FLUCTUATIONS IN LIQUID NITROBENZENE

V. S. Starunov, E. V. Tiganov, and I. L. Fabelinskii
P. N. Lebedev Physics Institute, USSR Academy of Sciences
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In all liquids, the spectrum of light scattered by density fluctuations consists of a linearly-polarized Rayleigh triplet, which fits within a frequency interval narrower than 1 cm^{-1}, and a continuous depolarized spectrum (the wing of the Rayleigh line), which extends to 150 cm^{-1} [1].

The frequency dependence of the intensity in the depolarized-scattering spectrum is divided roughly into two regions, one adjacent to the unshifted line (0 - 25 cm^{-1}) and the remaining region [1,2].

The region directly adjacent to the unshifted line - the narrow or diffuse wing, considered in detail by Starunov [2] - is characterized by a different width and intensity. Its maximum coincides with the unshifted line. In liquids with anisotropic molecules, which are relatively viscous at room temperature (glycerin, triacetin, salol, acetic acid, etc.) the diffuse wing is very narrow [1].

Of special interest is nitrobenzene - a liquid whose molecules have a very large anisotropy, and whose viscosity is ~2 centipoise. The thermal scattering spectrum of this liquid has not yet been investigated.

The use of a gas laser (λ = 6328 Å) as the light source affords a qualitatively different opportunity for simultaneously investigating the narrow diffuse wing and the fine-structure lines.

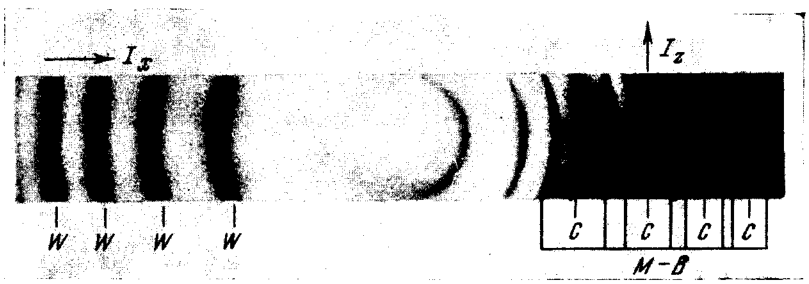

Fig. 1. Spectrum of thermal scattering of light in nitrobenzene. M-B - Mandel'shtam-Brillouin components, C - central component, W - diffuse wing. Fabry-Perot interferometer dispersion region - 0.50 cm^{-1}.

Figure 1 shows the thermal-scattering interference spectrum of nitrobenzene. The ap-