lead to the appearance in its spectrum of photons of relatively high energy, which do not appear when scattering is not taken into account [7]. Apparently a similar phenomenon takes place in resonance radiation, which is not taken into account in the theory.

The fact that there is no theory of resonance radiation with allowance for the influence of multiple scattering, together with the interference between the resonance and the bremsstrahlung radiations, does not permit a corresponding comparison between experiment and theory.

Thus, we have experimentally observed radiation of electrons in a layered medium, with an intensity that exceeds by many times in the x-ray region the intensity of the bremsstrahlung, and which depends strongly on the particle energy (like E^n, where $n \gtrsim 2$); this can be used to measure the particle energy.

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CONSEQUENCES OF CROSSING SYMMETRY FOR πN-SCATTERING S WAVES

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The problems of symmetry and invariance have recently gained in importance for the study of elementary particles. In this connection, Wigner [1] proposed a general classification of invariance properties, according to which all the properties are divided into geometric and dynamic. An intermediate position in the proposed classification is occupied by the principle of crossing symmetry, which can be formulated in the language of events, i.e., without reference to any particular form of interaction. However, his formulation and the experimental verification have made it possible to propose that an analytic expression for the amplitude of the investigated process is known. We present below the application of crossing symmetry to the analysis of elastic πN scattering. We shall use an approximate expression for the principle of crossing symmetry, viz., the static limit. In this approximation the crossing symmetry expresses the S wave of the $\pi^- p$ scattering process in terms of the S wave of the $\pi^+ p$ scattering process. The situation with the higher waves is similar. The most general properties of the solutions of the Chew-Low equations allow us to obtain for the matrix elements $S_j(\omega) = \exp[2i\delta_j(\omega)]$, as functions of the total meson energy $\omega = (1 + q^2)^{1/2}$, expressions that take into account the crossing-symmetry condition. These properties are: 1) $S_i(\omega)$ is a meromorphic function in the complex ω plane with cuts $(-\infty, -1]$ and $[+1, +\infty)$; 2) $S_i^*(\omega) = S_i(\omega^*)$; 3) $|Si(\omega + i0)|^2 = 1$, $\omega > 1$; 4) $S_i(-\omega) = A_{ij}S_j(\omega)$ and $A = \frac{1}{3}\begin{pmatrix}-1, & 4\\ 2, & 1\end{pmatrix}$. The fourth property is approximate. However, an estimate of the errors of the type $1/M$ (M = nuclear mass in meson-mass units) in the right side of 4) shows that they are < 30% for pion energies < 300 MeV in the lab. system. In the same region of energy and unitarity conditions, 3) is satisfied with great accuracy. Conditions (1) lead to analytic expressions [2] for $S_{1/2}(\omega)$ and $S_{3/2}(\omega)$:

$$S_{1/2}(\omega) = \frac{B(\omega)[B(\omega) - 2]}{B^2(\omega) - 1}D(\omega); \qquad S_{3/2}(\omega) = \frac{B(\omega)}{B(\omega) - 1}D(\omega);$$
$$B(\omega) = \frac{\omega P(\omega^2)}{qQ(\omega^2)} = \frac{1}{2} - \frac{1}{\pi}\ln(\omega + q); \qquad D(\omega) = \frac{1 + i\tan\Delta(\omega)}{1 - i\tan\Delta(\omega)}; \quad \tan\Delta(\omega) = q\frac{R(\omega^2)}{Q(\omega^2)}, \tag{2}$$

where P, Q, R, and G are polynomials of ω^2. The crossing symmetry condition 4) indicates how the parameters must be introduced in the different πN-scattering S waves. From the practical point of view, it allows us to transfer information on $\pi^+ p$ scattering to $\pi^- p$ scattering, a process less studied experimentally than $\pi^+ p$ scattering. A confirmation of the crossing symmetry condition in the form 4) should be the fact that the experimental data on the S waves are well described within the framework of formulas (2). For such a verification we used the experimental material on S phases up to 460 MeV energy given in McKinley's paper [3]. Application of the effective-radius theory to this material has not made it possible to describe well the S phases. Formulas (2) give the following results for the S phases:

$$\delta_i(\omega) = \Delta_i(\omega) + \Delta(\omega);$$
$$\tan\Delta_1(\omega) = \frac{4q/\omega\, B_1(\omega)}{4B_1^2(\omega) + 3(q/\omega)^2}; \qquad \tan\Delta_3(\omega) = -\frac{1}{2}\frac{q/\omega}{B_1(\omega)};$$
$$\tan\Delta(\omega) = q(b_0 + b_1q^2 + b_2q^4);$$
$$B_1(\omega) = a_0 + a_1q^2 + a_2q^4 + \frac{q}{\omega}\frac{1}{\pi}\ln(\omega + q), \tag{3}$$

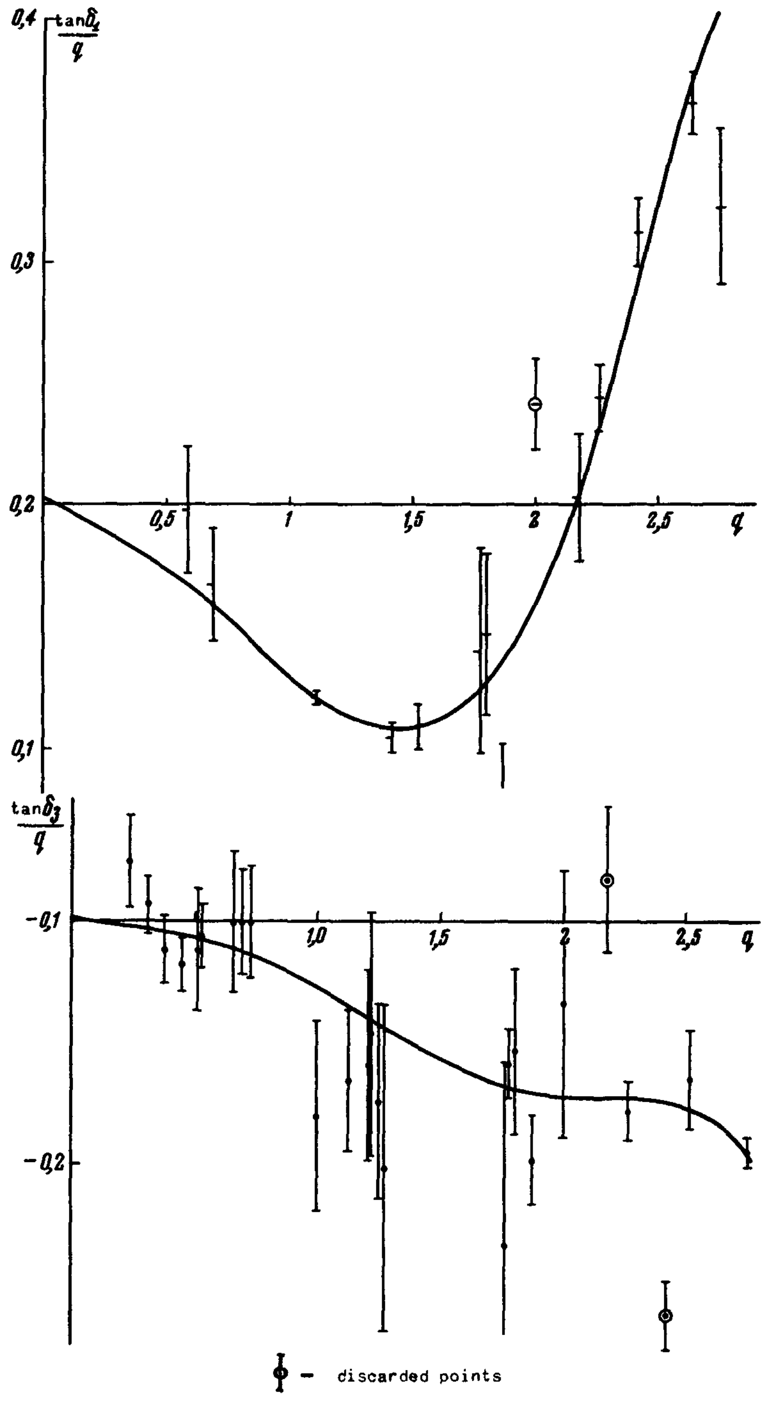
where

$$a_0 = 4.91 \pm 0.14,\ a_1 = -1.12 \pm 0.063,\ a_2 = 0.058 \pm 0.0063,$$
$$b_0 = 0,\ \ b_1 = 0.0495 \pm 0.0028,\ \ b_2 = 0.0095 \pm 0.00051$$

and χ^2/number of degrees of freedom = 1.015. From the experimental material we excluded three points: δ_1 - 270 MeV and δ_3 - 307 MeV [4] and 370 MeV [5]. The contribution of each of them to χ^2 exceeded 10 and their sum equalled the contribution of all the remaining 40 points. The scattering lengths are:

$$a_1 - a_3 = 0.305(1 \pm 0.03), \quad a_1 + 2a_3 = 0, \tag{4}$$

and agree with the latest experimental data [6]. Formulas (3) describe well the experimental data at 98, 150, and 170 MeV, which cause great difficulties in other methods (see figures).



Thus, the experimental data on the πN-scattering S phases confirm the deductions of the approximate crossing symmetry up to 460 MeV. On the other hand, the analysis presented offers an example of those consequences which can be duduced from the crossing symmetry conditions, and also points to ways of their experimental verification.

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SUPPRESSION OF SELF-FOCUSING OF LIGHT BEAMS AND STABILIZATION OF A PLANE WAVE IN A WEAKLY ABSORBING MEDIUM

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Bespalov and Talanov [1] have established that in a nonlinear transparent medium with a dielectric constant that depends on the electric field, $\epsilon = \epsilon_0 + \delta\epsilon$, $\delta\epsilon = \epsilon_2 E^2 > 0$, a plane light wave is unstable against small perturbations of the field and breaks up spontaneously into individual self-focusing beams [2,3].

As will be shown below, the presence of weak absorption in the medium affects strongly the time evolution of the processes of self-focusing and decay of the plane wave. Slight heating of the medium, which accompanies the absorption, leads to thermal expansion and to occurrence of a negative increment $\delta\epsilon_{th}$. Generally speaking, $\delta\epsilon_{th} = (\partial\epsilon/\partial\rho)_T \delta\rho_{th} + (\partial\epsilon/\partial T)_\rho \delta T$, where $(\partial\epsilon/\partial T)_\rho$ can be either positive or negative. In most cases, however, the role of the second term is small and we shall leave it out. Whereas the Kerr effect and electrostriction, which produce positive increments $\delta\epsilon_{Kerr,\ str}$, give rise to self-focusing of the light beam, the absorption exerts a defocusing action [1]. Owing to the constant increase in $|\delta\epsilon_{th}|$ with time (as heat is being released), the self-focusing effect must be suppressed after a certain time. Absorption, consequently, exerts a stabilizing effect on the plane wave (actually - on a parallel beam of large supercritical power), suppressing each case of instability flare-up after the lapse of sufficient time. Let us consider an isolated parallel light beam of radius R. We assume that the field in the beam decreases along the radius from the axis