Thus, the experimental data on the πN-scattering S phases confirm the deductions of the approximate crossing symmetry up to 460 MeV. On the other hand, the analysis presented offers an example of those consequences which can be duduced from the crossing symmetry conditions, and also points to ways of their experimental verification.

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SUPPRESSION OF SELF-FOCUSING OF LIGHT BEAMS AND STABILIZATION OF A PLANE WAVE IN A WEAKLY ABSORBING MEDIUM

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Bespalov and Talanov [1] have established that in a nonlinear transparent medium with a dielectric constant that depends on the electric field, $\epsilon = \epsilon_0 + \delta\epsilon$, $\delta\epsilon = \epsilon_2 E^2 > 0$, a plane light wave is unstable against small perturbations of the field and breaks up spontaneously into individual self-focusing beams [2,3].

As will be shown below, the presence of weak absorption in the medium affects strongly the time evolution of the processes of self-focusing and decay of the plane wave. Slight heating of the medium, which accompanies the absorption, leads to thermal expansion and to occurrence of a negative increment $\delta\epsilon_{th}$. Generally speaking, $\delta\epsilon_{th} = (\partial\epsilon/\partial\rho)_T \delta\rho_{th} + (\partial\epsilon/\partial T)_\rho \delta T$, where $(\partial\epsilon/\partial T)_\rho$ can be either positive or negative. In most cases, however, the role of the second term is small and we shall leave it out. Whereas the Kerr effect and electrostriction, which produce positive increments $\delta\epsilon_{Kerr,\ str}$, give rise to self-focusing of the light beam, the absorption exerts a defocusing action [1]. Owing to the constant increase in $|\delta\epsilon_{th}|$ with time (as heat is being released), the self-focusing effect must be suppressed after a certain time. Absorption, consequently, exerts a stabilizing effect on the plane wave (actually - on a parallel beam of large supercritical power), suppressing each case of instability flare-up after the lapse of sufficient time. Let us consider an isolated parallel light beam of radius R. We assume that the field in the beam decreases along the radius from the axis toward the edge. We represent the light flux density $I = \sqrt{\epsilon_0}cE^2/4\pi$ in the form $I(z, r, t) = I_0(t)f(r/R)$, where $I_0(t)$ pertains to the axis, t is the time from the instant of arrival of the light wave at the given section of the light channel z, and $f(\xi)$ is a profile function ($\xi = r/R \ll 1$, $f(0) = 1$), for example $f = \cos(\xi\pi/2)$. The change in density of the substance in a given section of the channel, $\delta\rho(r, t) = \rho - \rho_0$, is described by the linearized equations of hydrodynamics

$$\frac{\partial\delta\rho}{\partial t} + \frac{\rho_0}{r}\frac{\partial}{\partial r} ru = 0; \quad \rho_0\frac{\partial u}{\partial t} = - a^2\frac{\partial\delta\rho}{\partial r} - \frac{\partial p_{th}}{\partial r} - \frac{\partial r_{str}}{\partial r}, \tag{1}$$

$$p_{th} = \Gamma\kappa_\nu\int_0^t I dt; \quad p_{str} = - \rho\frac{\partial\epsilon}{\partial\rho}\frac{E^2}{8\pi}. \tag{2}$$

Here u is the radial velocity, a the speed of sound, p_{str} the striction pressure, p_{th} the rise in pressure of the substance, at the instant t, due to heat release without change of density, κ_ν the light absorption coefficient, and Γ the derivative of the pressure with respect to the internal energy per unit volume, taken at constant volume.

During the early stage, when $t \ll t_s = R/a$, we can leave out the term $-a^2\partial\delta\rho/\partial r$ from (1) with good approximation. Putting for simplicity $I_0(t) = \text{const}$, we get

$$\delta\rho\ \frac{\Psi}{6}\frac{\Gamma\kappa_\nu I_0 t^3}{R^2}(1 - 3\tau_{s0}/t) = \frac{\Psi}{6}\frac{p_{0\ th}}{a^2}(t/t_s)^2(1 - 3\tau_{s0}/t), \tag{3}$$

where the function $\Psi = f'' + f'/\xi$, just like f, decreases from the axis toward the edge; $p_{0\ th}$ pertains to the channel axis, and $\tau_{s0} = (\rho\partial\epsilon/\partial\rho)/2\Gamma\sqrt{\epsilon_0}c\kappa_\nu$. The first terms in (3) correspond to $\delta\rho_{th}$ and the second to $\delta\rho_{str}$. When $t \gg t_s$ the density is at quasi-equilibrium and

$$\delta\rho = - \frac{p_{th} + p_{str}}{a^2} = - \frac{p_{0\ th}}{a^2}(1 - \tau_{s0}/t)f = - \frac{\Gamma\kappa_\nu I_0 t}{a^2}(1 - \tau_{s0}/t)f. \tag{4}$$

The limiting formulas (3) and (4) can be approximately extrapolated to the instant $t = t_s$, where they join together fairly well. A time of the order τ_{s0} separates two stages: when $t < 3\tau_{s0}$ or $t < \tau_{s0}$, depending on the ratio of the times τ_{s0} and t_s, striction compression predominates and $\delta\rho > 0$, $\delta\epsilon_\rho = \delta\epsilon_{str} + \delta\epsilon_{th} = (\partial\epsilon/\partial\rho)\delta\rho > 0$. When $t > 3\tau_{s0}$ or $t > \tau_{s0}$, thermal expansion predominates and $\delta\rho$, $\delta\epsilon_\rho < 0$.

We now compare $\delta\epsilon_{th}$ and $\delta\epsilon_{Kerr}$. Bearing in mind that $\Psi/6f \approx 1$, we find approximately that $|\delta\epsilon_{th}|$ becomes larger than $\delta\epsilon_{Kerr}$, starting with an instant $t = \tau_k$ equal to

$$\tau_k = \tau_{k0} \times \begin{cases} (t_s/\tau_{k0})^{\frac{2}{3}}, & \text{if } t_s > \tau_{k0}, \\ 1, & \text{if } t_s < \tau_{k0}, \end{cases} \tag{5}$$

$$\tau_{k0} = 4\pi\rho_0 a^2\epsilon_{2Kerr}/\rho\frac{\partial\epsilon}{\partial\rho}\Gamma\sqrt{\epsilon_0}c\kappa_\nu.$$

We present numerical estimates. Let $\rho_0 = 1\ g/cm^2$, $a = 1$ km/sec, $\rho\partial\epsilon/\partial\rho = 1$, $\sqrt{\epsilon_0} = 1.5$, and $\Gamma = 2$, as is characteristic of liquids. Let $\kappa_\nu = 10^{-3}\ cm^{-1}$ and let the free path of light

$l_\nu = 1/\kappa_\nu = 10$ m. In addition, $\tau_{s0} = 5$ nsec. If $\epsilon_{2Kerr} = 10^{-12}$ abs. un. (the Kerr effect is weakly pronounced), then $\tau_{k0} = 1.4$ nsec. In all cases of practical interest $t_s > \tau_{k0}$ (R $> 1.4 \times 10^{14}$ cm), so that $|\delta\epsilon_{th}|$ increases to $\delta\epsilon_{Kerr}$ even before mechanical quasi-equilibrium is established and $\tau_k = \tau_{k0}^{1/3}\tau_s^{2/3}$. For example, $t_s = 10^3$ nsec and $\tau_k = 110$ nsec when R = 0.1 cm. The striction effect is suppressed much earlier, after $3\tau_{s0} = 15$ nsec. In the liquid which is most active from the point of view of the Kerr effect, namely carbon disulfide, $\epsilon_{2Kerr} = 6 \times 10^{-11}$ abs. un. At the same value $\kappa_\nu = 10^{-3}$ cm^{-1} we have $\tau_{k0} = 60$ nsec (we note that the absorption can be easily increased by adding absorbers). If $R < 0.7 \times 10^{-2}$ cm then $\tau_k = \tau_{k0} = 60$ nsec. When R = 0.1 cm we have $t_s = 800$ nsec and $\tau_k = 400$ nsec.

Thus, striction (owing to the slowness of the establishment of mechanical equilibrium) yields a $\delta\epsilon_{str}$ much smaller than $\delta\epsilon_{Kerr}$ and for an absorption $\kappa_\nu = 10^{-3}$ cm^{-1} the effect of striction is quite rapidly suppressed by the effect of thermal expansion. The Kerr effect exerts a focusing action much longer, but even so it becomes neutralized after a time τ_k by the defocusing action of the thermal expansion. When $t > \tau_k$ the divergence of an isolated light beam increases continuously with time. The time $\tau_k \sim 10^{-7}$ sec is much longer than the duration of a giant laser pulse, but it is shorter than the duration of individual beams from solid-state lasers operating in the free-generation mode, thus uncovering possibilities for an experimental observation of the suppression of self-focusing.

The foregoing considerations are fully applicable also to investigations of the decay of a plane wave. The thermal action of the absorption will incessantly suppress each time the instability flare-up, rectifying the beams and equalizing in the transverse direction the field between the individual self-focusing beams. The duration of the "flare-ups" will be determined by the time τ_k. We note that formally, too, the negative addition $\delta\epsilon = \epsilon_2 E^2 < 0$ exerts a stabilizing action on a plane wave, for the perturbations do not grow in this case. This can be readily verified by repeating the calculations of [1] with $\epsilon_2 < 0$.

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