pressions for two-particle processes in quantum electrodynamics.

The authors thank I. A. Malkin, I. Ya. Pomeranchuk, and E. S. Fradkin for useful discussions.

[1] I. C. Polkinghorne, J. Math. Phys. 4, 503 (1963).
[2] V. V. Sudakov, JETP 30, 87 (1956), Soviet Phys. JETP 3, 65 (1956).
[3] J. D. Bjorken and T. T. Wu, Phys. Rev. 130, 2566 (1963).
[4] A. A. Abrikosov, JETP 30, 386, 544 (1956), Soviet Phys. JETP 3, 474, 379 (1956).
[5] G. A. Milekhin and E. S. Fradkin, JETP 45, 1926 (1963), Soviet Phys. JETP 18, 1323 (1964).
[6] G. V. Frolov, V. G. Gorshkov, and V. N. Gribov, Phys. Lett. 20, 544 (1966).
[7] V. N. Baier and S. A. Kheifets, JETP 40, 613 (1961), Soviet Phys. JETP 13, 428 (1961).
[8] D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. of Phys. 13, 379 (1961).

1) The authors thank I. A. Malkin for calling their attention to this reaction.

## CONSISTENT RELATIVIZATION OF AN SU(6) GROUP FOR TWO-PARTICLE REACTIONS

V. A. Ogievetskii and I. V. Polubarinov
Joint Institute for Nuclear Research
Submitted 10 July 1966
ZhETF Pis'ma 4, No. 8, 325-329, 15 October 1966

1. So far, no one has succeeded in finding a relativistic version of the SU(6) group with internal consistency, i.e., one leaving invariant the free equations and leading to noncontradictory limitations on the reaction amplitudes. The only success in this direction was attained for collinear processes, for which the group $SU_W(6)$ was found [1].

We propose here a group $SU_x(6)$, isomorphic to SU(6), satisfying the foregoing requirements, and applicable to two-particle reactions without being confined to collinearity [1)].

2. Let us examine this group with quarks as the example. For quarks, the corresponding transformations are written in the form

$$\delta\Psi(p) = [\tfrac{i}{2}\omega^a\lambda_a + e^i_\mu(p)\gamma_\mu\gamma_5(\alpha_i + \alpha^a_i\lambda_a)]\Psi(p), \tag{1}$$

where $e^i_\mu(p)$ are three vectors that are orthogonal to one another and to the momentum p: $(e^i\cdot p) = 0$, $(e^i\cdot e^j) = \delta_{ij}$, $\lambda_a$ are 8 Gell-Mann matrices, and $\omega^a$, $\alpha_i$, and $\alpha^a_i$ are the transformation parameters. These transformations commute with the free Dirac equation. The vectors $e^i_\mu(p)$ can be expressed in many ways in terms of the momenta of the particles that participate in the reactions. For two-particle reactions, we think that the physically most natural choice of basis is:

$$e^1_\mu(p) = N_1(P_\mu - \frac{p^2}{(Pp)}p_\mu),\ e^2 = N_2\epsilon_{\mu\nu\lambda\rho}p_{1\nu}p_{2\lambda}p_{3\rho},\ e^3_\mu(p) = N_3\epsilon_{\rho\nu\lambda\rho}e^1_\nu e^2_\lambda p_\rho, \tag{2}$$

where $P_\mu = p_{1\mu} + p_{2\mu} = p_{3\mu} + p_{4\mu}$ is the total momentum, and $N_1$, $N_2$, and $N_3$ are normalization factors. It is convenient to go over (without loss of the relativistic meaning) to a c.m.s. ($\vec{P} = 0$) and to two-component spinors $\varphi(\vec{p})$. Then (1) takes the form

$$\delta\varphi(\vec{p}) = \left\{\frac{i}{2}\,\omega^a\lambda_a + i(\alpha_1 + \alpha_1^a\lambda_a)\frac{(\vec{\sigma}\cdot\vec{p})}{|\vec{p}|} + i(\alpha_2 + \alpha_2^a\lambda_a)(\vec{\sigma}\cdot\vec{n}) + i(\alpha_3 + \alpha_3^a\lambda_a)\frac{(\vec{\sigma}\cdot\vec{n}\times\vec{p})}{|\vec{p}|}\right\}\varphi(\vec{p}), \tag{3}$$

where $\vec{n}$ is a unit vector normal to the reaction plane. It is interesting that the operator $ie_\mu^1(p)\gamma_\mu\gamma_5$ turns out to correspond to the helicity $(\vec{\sigma}\cdot\vec{p})/|\vec{p}|$. This means that the total helicity is conserved in the invariant amplitudes.

3. For any momentum, the transformations (3) are isomorphic to the ordinary momentum-independent transformations

$$\delta\varphi'(\vec{p}) = \left\{\frac{i}{2}\,\omega^a\lambda_a + i(\alpha_k + \alpha_k^a\lambda_a)\sigma_k\right\}\varphi'(\vec{p}). \tag{4}$$

Indeed, the transformations (3) and (4) are related by the similarity transformation

$$\varphi'(\vec{p}) = S(\vec{p})\varphi(\vec{p}), \tag{5}$$

where the matrix $S(\vec{p})$ effects spin rotation, which transforms $(\vec{\sigma}\cdot\vec{p})/|\vec{p}|$ into $\sigma_x$, $(\vec{\sigma}\cdot\vec{n})$ into $\sigma_y$, and $(\vec{\sigma}\cdot\vec{n}\times\vec{p})/|\vec{p}|$ into $\sigma_z$, but does not affect the momenta themselves. We choose $S(\vec{p})$ in the form

$$S(\vec{p}) = \exp\left(\frac{i}{2}\,\vec{\omega}\cdot\vec{\sigma}\right), \quad \vec{\omega} = -\vec{n}\arcsin\frac{p_z}{|p|}\,. \tag{5'}$$

Quarks with different momenta transform in accordance with different but equivalent representations of the SU(6) group, and the similarity transformation (5) makes them all fully equivalent.

4. The transition to the spinors $\varphi'(\vec{p})$ makes it easy to construct the invariant amplitudes. Since (4) simply coincide with the ordinary transformations of SU(6) and do not depend on the momenta, the invariant amplitudes are constructed in these terms in the c.m.s. in accordance with the usual rules of SU(6). Thus, the amplitude for the scattering of a quark is written in the form

$$A(\varphi_4'^{+}\varphi_2')(\varphi_3'^{+}\varphi_1') + B\,(\varphi_4'^{+}\varphi_1')(\varphi_3'^{+}\varphi_2'); \quad (\varphi_i' = \varphi'(\vec{p}_i)),$$

where A and B are arbitrary form factors that depend on the energy and on the scattering angle $\theta$. In the languate of ordinary spinors the amplitude has a more complicated form, for example

$$(\varphi_4'^{+}\varphi_2')(\varphi_3'^{+}\varphi_1') = \cos^2\frac{\theta}{2}(\varphi_4^{+}\varphi_2)(\varphi_3^{+}\varphi_1) - \frac{i}{2}\sin\theta[(\varphi_4^{+}(\vec{\sigma}\cdot\vec{n})\varphi_2)(\varphi_3^{+}\varphi_1) + (\varphi_4^{+}\varphi_2)(\varphi_3^{+}(\vec{\sigma}\cdot\vec{n})\varphi_1]$$

$$- \sin^2\frac{\theta}{2}(\varphi_4^{+}(\vec{\sigma}\cdot\vec{n})\varphi_2)(\varphi_3^{+}(\vec{\sigma}\cdot\vec{n})\varphi_1).$$

It is logical to work directly in the terms of the "primed" spinors (5).

The situation with other multiplets (35, 56, etc.) is similar [2]. There, too, it is possible to go over with the aid of spin expressions similar to (5) to new ("primed") quantities which transform in accordance with ordinary SU(6). Then the amplitudes are also constructed in accordance with the SU(6) group rules.

5. The matrices $S(\vec{p})$ drop out of the cross sections summed over the spin states. Indeed, $\sum_s \varphi'(s)\varphi'^{+}(s) = S \sum_s \varphi(s)\varphi^{+}(s)S^{-1} = SS^{-1} = 1$. Thus, we reach the important conclusion that all the consequences for the cross sections summed over the polarizations do not depend on the choice of the vectors $e^{i}_{\mu}$. These consequences are the same as if we were to construct the cross sections in accordance with the rules of SU(6) without paying attention to the fact that the momenta differ from zero. The correctness of the choice of the basis can be verified only by an evaluation of the polarization effects.

6. Since the obtained group is an internal symmetry group, which commutes with the free equations, the question of elastic unitarity does not arise. Thus, in the simple case of scattering of a unitary singlet by a quark [3], the scattering amplitude takes the form $A(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}(\vec{\sigma}\cdot\vec{n}))$, meaning that the connection between the phases is $\delta^{-}_{e+1} = \delta^{+}_{e}$, i.e., for a given total angular momentum j the phases are degenerate in the orbital angular momentum $l$.

7. At the same time, the obtained group $SU_x(6)$ can apparently be employed only at very high energies, which level out the mass differences of the states that enter in one multiplet, just as the use of the isotopic group is justified only at energies exceeding the electromagnetic mass splitting. At low energies $SU_x(6)$ may lead to strong deviation from experiment.

Thus, we have obtained a dynamic (just like $SU_W(6)$) relativistic group $SU_x(6)$, which commutes with the equations of motion, is internally consistent, and is isomorphic to SU(6). The consequences of this group and the resultant predictions will be considered elsewhere.

The authors are sincerely grateful to A. N. Zaslavskii, B. L. Ioffe, I. Yu. Kobzarev, M. A. Markov, B. V. Struminskii, A. N. Tavkhelidze, and V. Tybor for a discussion and remarks.

[1] H. I. Lipkin and S. Meshkov, Phys. Rev. Lett. 14, 670 (1965);
R. F. Dashen and M. Gell-Mann, Phys. Lett. 17, 145 (1965);
K. J. Bernes, Phys. Rev. 139B, 947 (1965).

[2] V. I. Ogievetskii and I. V. Polubarinov, JINR Preprint R-2698; YaF 4, 853 (1966), Soviet JNP 4, in press.

[3] B. V. Geshkenbein, B. L. Ioffe, M. S. Marinov, and V. I. Roginskii, JETP Letters 1, No. 6, 23 (1965), transl. 1, 164 (1965).

[1] This group arose as a subgroup of the infinite-parametric group which we have considered earlier; the $SU_W(6)$ group is a particular case of $SU_x(6)$.

[2] The transformations for the 35-, 56-, and 189-plets can be found in the authors' paper [2] (see Sec. 5 and Appendix 4).