THE TREIMAN-YANG CRITERION FOR PARTICLES WITH SPIN

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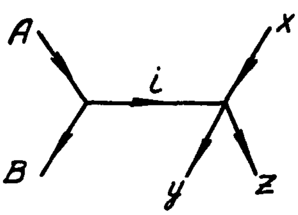
If a reaction proceeds via exchange of a zero-spin particle then, as is well known, its pole character can be verified with the aid of the Treiman-Yang criterion [1]. It was shown earlier [2] that for nonrelativistic particles there exist a number of cases when the Treiman-Yang criterion is applicable in spite of the fact that the spin j_i of the pole particle differs from zero (in particular, when $j_i = 1/2$). In recently published articles [3,4] the authors state that the Treiman-Yang criterion is satisfied also in the relativistic case when a particle with spin 1/2 is exchanged. We shall show below that: 1) this statement is incorrect; 2) a number of cases when the Treiman-Yang criterion is satisfied, given in [2], remain valid for nuclear reactions at high energies, when the left-hand vertex of the diagram of the figure is nonrelativistic and the right-hand one is relativistic.



Pole diagram

1. The satisfaction of the Treiman-Yang criterion in the nonrelativistic case with $j_i = 1/2$ is connected with the fact that, first, a particle with spin 1/2 is not aligned in terms of the higher polarization moments and, second, the spin wave functions do not change under Galilean transformations (there is no relativistic spin rotation). A simple example shows that for a relativistic particle the situation is different. Let us calculate in accordance with the Feynman rules the amplitude M of the reaction

$$A + x \to B + y + z, \tag{1}$$

corresponding to the diagram of the figure, if A, i, and y are particles with spin 1/2 and mass m, while B, x, and z are scalar particles. Let the coupling constants in the left-hand vertex be g, and let the amplitude of the process $i + x \to y + z$ be $ia\hat{q} - b$; a and b are invariant form factors. Then [5]

$$M = i \frac{(2\pi)^4 g}{(2\omega_B)^{\frac{1}{2}}} \bar{u}_z(ia\hat{q} - b) \frac{i\hat{p}_i - m}{p_i^2 + m^2} u_A. \tag{2}$$

Here u_z and u_A are the spinors corresponding to particles z and A, normalized such that u^+u

$= p^0/m$. ω_B denotes the energy of particle B. To simplify the formulas we assume that g, a, and b are real quantities. We denote by F the square of the modulus of the amplitude of reaction (1), summed over the spin states of the final particles and averaged over the spin states of the initial particles. By F_1 and F_2 we denote the analogous quantities for the reactions $A \to B + i$ and $i + x \to y + z$. We note that F_1 is a function of only the variable $t = -(p_B - p_A)^2$, and F_2 depends on t, $s' = (p_y + p_z)^2$, and $t' = (p_z - p_x)^2$. From (2) we obtain

$$\frac{p_i^2 + m^2}{(8\pi)^2} F = 4m^2F_1F_2 + \frac{g^2(p_i^2 + m^2)}{4m^2\omega_B} [-(p_Ap_y)(b^2 + a^2q^2) + 2a^2(p_Aq)(p_yq) + 2abm(p_yq) + m^2(a^2q^2 - b^2) - 2abm(p_Aq)]. \quad (3)$$

Thus, besides the terms that depend on t, t', and s', expression (3) for F contains also terms of the form (p_Ap_y) and (p_Aq), which depend on $t_{Ay} = -(p_A - p_y)^2$ and $s = -(p_A + p_x)^2$. F will be changed by a Treiman-Yang rotation, being a linear function of $\cos\varphi$ (φ is the Treiman-Yang angle). A similar situation obtains even if we leave only the scalar part in the amplitude of the process $i + x \to y + z$ (i.e., if a = 0), and consequently in the case when the reaction $i + x \to y + z$ is resonant. (These are just the reactions considered in [3,4].)

2. The terms that depend on t_{Ay} and s and which prevent the separation of F into two factors (F_1 and F_2) contain $(p_i^2 + m^2)$ and in the case of a nonrelativistic left-hand vertex they have an order of smallness

$$\frac{p_i^2 + m^2}{m^2} \sim (v_i/c)^2 \ll 1 \quad (4)$$

regardless of whether the right-hand vertex is relativistic or not. If condition (4) is satisfied, then the Treiman-Yang criterion will be applicable for arbitrary energies of the particles x, y, and z in the following three cases: (a) $j_i = 0$ or 1/2; (b) j_i is arbitrary, but $l_{iB} = 0$, where l_{iB} is the orbital angular momentum of the relative motion of the particles i and B; (c) $j_{iB} = 0$ or 1/2, where j_{iB} is the total spin of the particles i and B.

We note that the distribution with respect to the Treiman-Yang angle should be symmetrical about $\varphi = 0$, regardless of the diagram describing the amplitude of reaction (1). This follows from the invariance against reflection in the plane of the momenta of the particles A, B, and i in the antilaboratory system (the system in which $\vec{p}_x = 0$).

Symmetry in the distribution relative to the Treiman-Yang angle is mentioned in [3], where it is expressed by the relation $F(\varphi = 0) = F(\varphi - \pi)$. Formulas (17) - (20) of [2] show that this relation does not hold true, since the expression for F contains both even and odd powers of $\cos\varphi$. This can be illustrated by means of a simple example. Let the particles A, B, x, y, and z be scalar, and let the particle i be pseudovector (y and z need not be identical). We introduce unit vectors $\vec{n}$, $\vec{k}$, and $\vec{l}$, directed along the relative velocities of the particles B and i, i and x, and y and z. The amplitude of the left vertex of the diagram of the figure is $a(\vec{S}\cdot\vec{n})$, and that of the right vertex $[b(\vec{S}\cdot\vec{k}) + c(\vec{S}\cdot\vec{l})]$. For F we have

$$F = |a|^2|b|^2(\vec{n}\cdot\vec{k})^2 + |a|^2|c|^2(\vec{n}\cdot\vec{l})^2 + |a|^2(bc^* + b^*c)(\vec{n}\cdot\vec{k})(\vec{n}\cdot\vec{l}). \qquad (5)$$

$(\vec{n}\cdot\vec{k})$ and $(\vec{k}\cdot\vec{l})$ are not changed by the Treiman-Yang rotation, while $(\vec{n}\cdot\vec{l})$ is connected with $\cos\varphi$ by

$$(\vec{n}\cdot\vec{l}) = (\vec{n}\cdot\vec{k})(\vec{k}\cdot\vec{l}) + [(1 - (\vec{n}\cdot\vec{k})^2)(1 - (\vec{n}\cdot\vec{l})^2)]^{\frac{1}{2}}\cos\varphi. \qquad (6)$$

Therefore F (see (5)) contains $\cos\varphi$ in both the first and second degree.

In general, it can be stated that in the nonrelativistic case the quantity F, which corresponds to the diagram of the figure with arbitrary spin j_i, is a polynomial of degree n in $\cos\varphi$, with $n \leq \min\{2l_{iB}, [j_{iB}], [j_i], [j_{ix}], 2L\}$, where $[j] = 2j$ when j is integer and $[j] = 2j - 1$ when j is half-integer; j_{ix} is the total spin of the particles i and x. L is the geometric difference of the spins of the input (j_{ix}) and output (j_{yz}) channels of the reaction $i + x \to y + z$ (see [2]), and $|j_{ix} - j_{yz}| \leq L \leq (j_{ix} + j_{yz})$. The statement formulated above follows from relation (6) of the present note and from formulas (17) - (20) of [2]. In the relativistic case we can only state that $n \leq 2j_i$, for both integer and half-integer j_i [6].

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