a frequency 0.03 Hz).

3. In the region when the current-source power is linear in the field amplitude, the effective resistance is independent of the current flowing through the sample, within the limits of experimental accuracy (the current ranged from 2 to 20 A in the experiments).

4. The effective value of the resistance does not depend on the waveform of the field pulses and is determined exclusively by the amplitude of the variation of the field intensity. Figure 3 shows a characteristic example of the attenuation of the current in a superconducting loop for almost rectangular and triangular field pulses. Rectangular pulses of two different types of greatly differing rms field intensities were used. The amplitude of the variation of the field intensity was 6 kOe.

5. Measurements of the amount of evaporated helium has shown that the heat loss replenished by the current source does not exceed, in order of magnitude, 10% of the heat loss replenished by the field generator. The heat loss connected with the field generator does not experience a jump when the field amplitude drops below the threshold value.

The most closely related to our investigation is the work by Taquet [1], who observed the occurrence of resistance in a wire sample of a non-ideal superconductor of the second kind upon change of the external magnetic field. The presence of a threshold value of the amplitude of field variation was not established. The obtained experimental material did not enable Taquet to classify the occurrence of the resistance as an essentially new phenomenon different from the hitherto observed resistive states.

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[1] B. Taquet, J. Appl. Phys. 36, 3250 (1965).

BENDING OF TRAJECTORIES OF ASYMMETRICAL LIGHT BEAMS IN NONLINEAR MEDIA

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In the study of the self-action of light beams in nonlinear media, the interest of the researches, starting from the trail-blazing work [1 - 3], has been focused mainly on the phenomenon of self-focusing and self-trapping of the light (cf., e.g., [4]). We can point out, however, at least one more interesting case of self-action of light, resulting from nonlinear refraction, namely the bending and "twisting" of trajectories of asymmetrical light beams in media showe refractive index depends on the field intensity. Indeed, unlike self-focusing, when the intensity is symmetrically distributed over the beam cross sections and all the rays tend to be gathered at the center of the beam (where the refractive index is maximal) as a result of the nonlinear refraction, in the case of asymmetrical distribution the beam will bend as a whole in the direction in which the refraction is maximal.

In the geometrical optics approximation, the radius of curvature of the beam trajectory at each point and the optimal distribution (from the point of view of appearance of the "pure" bending effect) of the intensity over the cross section follow directly from the fundamental formula of geometric optics:

$$1/R = \vec{N}(\nabla n/n), \tag{1}$$

where R is the radius of curvature of the trajectory, $\vec{N}$ the unit vector of the principal normal, and n the refractive index (determined in our case by the field intensity at the given point, $n = n(E^2)$). Regarding as "pure" rotation the case when all the rays of the beam at any fixed beam cross section rotate about one axis, which is characteristic of the given cross section, and consequently $\nabla = -\vec{N}(d/dR)$, we get from formula (1)

$$n(R) = \text{const}/R. \tag{2}$$

From (2) we get the form of the field-intensity distribution over the cross section, $E^2(R)$, in the case of "pure" rotation, if we specify the concrete form of $n(E^2)$. Let us specify $n(E^2)$ in the form $n = n_0 + n_2E^2$ and consider a two-dimensional light beam of <u>limited</u> cross section. By virtue of the condition $n_0 \gg n_2E^2$, which is usually satisfied in optics, the transverse dimension of the beam is much smaller than the characteristic dimensions connected with the nonlinearity (in particular, also, the radius of curvature). Therefore the optimal intensity distribution (2) can be written in this case in the form

$$E^2_{opt}(z) \simeq \begin{cases} E_\perp^2 + \Delta E^2\left(1 - \frac{2z}{a}\right), & -\frac{a}{2} < z < \frac{a}{2} \\ 0, & |z| > \frac{a}{2} \end{cases}, \tag{3}$$

where z is the direction of the normal to the beam axis (z is reckoned in a direction opposite to the rotation axis), a is the transverse dimension of the beam ($a \ll R$), and ΔE^2 is the most important parameter of beam asymmetry (from the point of view of trajectory rotation). The radius of curvature of the trajectory of the entire beam, R, is determined in accord with (2):

$$R = \frac{n_0 a}{n_2 \Delta E^2} \tag{4}$$

It is obvious that formulas (2) and (4) are valid also for a three-dimensional beam, in the cross section of which the light intensity is independent of the coordinate along the rotation axis (wherever the intensity differs from zero). If such a beam has a cross section in the form of a rectangle with sides a and b, and the field in it is determined only by the asymmetrical component ($E_1^2 = 0$), then the total beam power equals $P = (n_0c/8\pi)ab\Delta E^2$, and formula (4) can be written in the form

$$R = \frac{n_0^2 c a^2 b}{8 n_2 \pi P}. \tag{5}$$

For the case $a = b = 0.5$ mm, $P = 10$ MW, and $n_2 = 9 \times 10^{-12}$ cgs esu (hydrogen sulfide), this amounts to $R \sim 300$ cm. At a cell length $\ell = 30$ cm, the angular deviation of such a beam from its initial direction is $\phi \simeq \ell/R \sim 6°$.

This angular deviation at the exit from a nonlinear medium can be used for angle scanning of the light beam by manipulating its input power.

Even more interesting effects should appear in the case when the light beam is not a plane wave but a converging one (say one focused from the outside or a self-focused beam). Let us consider, for example (in the geometric-optics approximation) the case of a converging beam focused in a medium by an external lens and having an amplitude profile given by (3). If we assume that the beam is very thin, i.e., that $a \ll f$ (where f is the distance from the point of entry of the beam into the medium to the focus), then the radius of curvature of the trajectory at each point is determined as before by formula (5), where we must put $a = a_0(1 - (s/f))$ and $b = b_0(1 - (s/f))$; here a_0 and b_0 are the transverse dimensions of the beam on entering the medium and s is the length of the trajectory from the entry point to the given point. (These relations determine the decrease of the beam cross section on approaching the focus.) Formula (5) then takes the form

$$R = R_0(1 - (s/f))^3, \tag{6}$$

where $R_0 = (n_0^2 c a_0^2 b_0)/(8 n_2 \pi P)$ is the radius of curvature on entering the medium. Since $R = ds/d\phi$, where ϕ is the angle of inclination of the tangent to the beam trajectory, we get by integrating (6)

$$s/f = 1 - \alpha^{1/2}/[2(\phi - \phi_\infty)]^{1/2}, \tag{7}$$

where $\alpha = f/R_0$ and $\phi_\infty = -\alpha/2$ (if we assume that $\phi = 0$ when $s = 0$). From this, taking (6) into account, we get an expression for R as a function of ϕ

$$R/f = \alpha^{1/2}/[2(\phi - \phi_\infty)]^{3/2}. \tag{8}$$

We see thus that the trajectory of the focused beam ($s > 0$) is a spiral rolling into a point. When $s < 0$ Eqs. (6) and (8) describe the trajectory of a diverging beam in a nonlinear medium; it is seen from (7) and (8) that this is a curve whose asymptote has an inclination angle ϕ_∞.

If $E^2(z)$ has besides the linear component (3) also a quadratic component

$$E_0^2 = \Delta E_0^2\left(1 - \frac{2z}{a_0}\right) + \delta E_0^2\left(1 - \frac{4z^2}{a_0^2}\right),$$

then self-focusing due to this component takes place in the beam. The rate of the self-focusing (assuming the wave front on entering the medium is plane) is given by [4]

$$a(s) = a_0\sqrt{1 - \left(\frac{x}{f_c}\right)^2},$$

where

$$f_c = \frac{a_0}{2}\sqrt{\frac{n_0}{n_2 \delta E_0^2}}$$

is the length of the nonlinear "focal distance." From this we get in accord with (5) (with allowance for the fact that $b/b_0 = a/a_0$)

$$R_c = R_0[1 - (x/f_c)^2]^{3/2}. \tag{9}$$

This equation corresponds to the following $R(\phi)$ dependence:

$$R_c/f_c = \alpha^2/(\alpha^2 + \phi^2)^{3/2}.$$

It is seen from (10) that the rate of winding of the trajectory into a spiral is much faster for self-focusing than in the case of external focusing with the same values of R_0 and f, as is to be expected.

The foregoing formulas, which describe the behavior of the trajectory of the beam as a whole, are valid, as already indicated, only in the geometric-optics approximation. On the other hand, the presence of diffraction leads to a distortion of the initial amplitude profile of the beam, and as a result to a deviation from the calculated trajectories (circle for the case of an unfocused beam or a spiral in the case of focusing). For example, in the former case, owing to the smoothing and diffraction-broadening of the amplitude profile, the radius of curvature of the entire beam increases continuously along the beam until its trajectory becomes a straight line (the beam can go over into a self-focused filament if the input power is sufficiently high). Assuming that the anomalous structure of the profile (3) is destroyed almost completely over a distance equal to the diffraction length of the beam $\ell_d \sim n_0 k_0 a^2$, where $k_0 = 2\pi/\lambda_0$ is the wave number in vacuum (we assume here that a = b), we can approximately estimate the angle of inclination of this line to the intial beam direction at the point of its entry into the medium, namely

$$\phi_m \sim \frac{\ell_d}{R_0} = \frac{2\pi a}{\lambda_0} n_2 \Delta E^2 = \frac{16\pi^2 n_2 P}{\lambda_0 n_0^2 c a} . \qquad (11)$$

For the example presented above, this angle amounts to $\sim 60°$. On the other hand, in the case of focusing, the diffraction that disrupts the initial profile of the beam should apparently cause the spiral, which winds towards the center, to start "unwinding" starting with some of its turns, until it turns into a straight line.

In general, it should be noted that in the phenomena under consideration, from the theoretical point, the diffraction does not play the same role as in self-focusing, namely, it does not lead to a threshold for the process. It is easy to see that the bending of the trajectories of asymmetric light beams in nonlinear media will occur at all input powers. It should also be noted that whereas for realization of self-focusing it is necessary to satisfy the condition $n_2 > 0$, the sign of n_2 does not play any role in the bending of the beam trajectory, since it determines only the direction of the deflection of the beam (when $n_2 < 0$ the beam is deflected towards the minimal field intensity); this makes it possible to use strong "thermal" nonlinearities.

[1] G. A. Askar'yan, Zh. Eksp. Teor. Fiz. 42, 1567 (1962) [Sov. Phys.-JETP 15, 1088 (1962)].
[2] V. I. Talanov, Izv. VUZov, Radiofizika 7, 564 (1964).
[3] R. Chiao, E. Garmire, and C. Townes, Phys. Rev. Lett. 13, 479 (1964).
[4] S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov, Usp. Fiz. Nauk 93, 19 (1967) [Sov. Phys.-Usp 10, 609 (1968)].