Let us see now how this estimate changes in the case when the aforementioned bound state is produced ($T < T_c$). Using the Abrikosov procedure and applying it formally to the problem with $S = 1/2$, we can easily show that

$$S^{-+}(\omega) = 2\pi Q^{-1} \int_{-\infty}^{+\infty} g_{1/2}(\omega' + \omega) g_{-1/2}(\omega') f(\omega')[1 - f(\omega' + \omega)] d\omega', \tag{3}$$

where $f(\omega) = (e^{\omega/T} + 1)^{-1}$, and the spectral density of the fermion pseudoparticles is

$$g_{\pm 1/2}(\omega) = \frac{1}{\pi} \frac{\Delta}{(\omega \pm \omega_{0/2})^2 + \Delta^2}, \quad \omega_0 \ll \Delta.$$

Here Δ denotes the binding energy of the electron-impurity pair, and the factor Q^{-1} in (3) "cancels" the presence of unphysical impurity states. It is easy to verify that in the case below $Q = 1/2$.

Inasmuch as $\omega_{mm'} \ll (\omega_0, T, \Delta)$, we find that when $(\omega_0, T) \ll \Delta$ the probability of reorientation of the nuclear spin of the impurity atom is

$$W_N \simeq A^2 T/\Delta^2. \tag{4}$$

This differs appreciably from expression (2), for now W_N is practically independent of the external magnetic field. It is easy to understand the cause of the change occurring in the character of the dependence of the rate of nuclear relaxation on ω_0. Upon formation of bound states, the width of the spectrum of the impurity electron-spin fluctuations is determined by the binding energy $\Delta \gg 1/\tau$, and when $\omega_0 \simeq T \ll T_c$ the earlier strong-field condition ($\omega_0 \tau \gg 1$) changes into the weak-field condition ($\omega_0/\Delta \ll 1$). This suppresses the dependence of the nuclear relaxation rate on ω_0.

When the temperature is gradually increased ($T \to T_c$) the rate of relaxation of the spins of the impurity nuclei increases and can greatly exceed the value given by formula (2) in the region $\Delta(T) \simeq \omega_0$. When the temperature goes through T_c and increases further, the impurity relaxation rate should gradually increase to its"normal" value given by (1). The region $T \gtrsim T_c$ deserves a special investigation (which is beyond the scope of the calculation scheme employed above).

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FIELD STATISTICS IN PARAMETRIC LUMINESCENCE

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A large number of recent theoretical and experimental papers are devoted to the process of parametric luminescence (PL) - the scattering of intense monochromatic light (pump) by a crystal with quadratic polarizability (see [1], where other references are cited). In the

present paper we discuss problems in the statistics of the field and of the photocounts in parametric luminescence.

1. The PL process in a nondissipative medium should satisfy the Manly-Rowe relations. In quantum language this corresponds to a conservation law [2] that reduces to the statement that the pump photons break up into pairs of scattered quanta. A correlation then arises between the energies (numbers of quanta) emitted in the parametrically-coupled modes. In the simple luminescence (and not superradiance) regime, when the number of luminescent quanta per mode is $\langle n_i \rangle \ll 1$, this correlation is much stronger than called for by the classical theory of the electromagnetic field. We shall illustrate this with an example. In the known model of two-mode parametric interactions [2] (modes a and b) we have in generation from the vacuum state (here ω_p and ϕ_p are the pump frequency and phase):

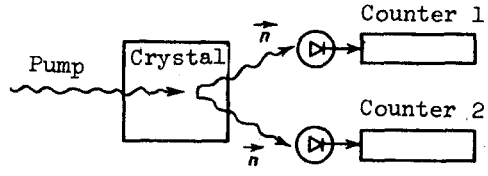
$$\langle a^+ a \rangle = G_{aa}(t) = G_{bb}(t) = \mathrm{sh}^2 kt, \quad \langle a^+ b \rangle = G_{ab} = G_{ba} = 0$$
$$\langle a b \rangle = B_{ab}(t) = \exp(-i\omega_p t - i\phi_p)\,\mathrm{sh}kt\,\mathrm{ch}kt. \tag{1}$$

When $kt \ll 1$ we have $\langle n_a \rangle G_{aa} \ll 1$ and

$$|\langle a b \rangle| \approx kt \gg \sqrt{\langle a^+ a \rangle \langle b^+ b \rangle} \approx (kt)^2 \tag{2}$$

whereas in the classical theory we obtain a rigorous inequality of opposite sign.

2. To observe this correlation, we propose the experiment illustrated in the figure.



In the case of simple lumnescence we can obtain explicit expressions for the joint distributions of the photocounts $p(n_1, n_2)$ or for the generating function $Q(\lambda_1, \lambda_2) = \Sigma p(n_1, n_2)(1 - \lambda_1)^{n_1}(1 - \lambda_2)^{n_2}$. For example,

$$Q(\lambda_1, \lambda_2) = \exp(-\lambda_1 \mu_1 - \lambda_2 \mu_2 + \lambda_1 \lambda_2 \mu_{12}). \tag{3}$$

Here the parameters μ_1 and μ_2 describe the mean number of detector counts, and μ_{12} their correlation. For the simplest case of luminescence in a flat nondissipative crystal we have

$$\mu_1 = T\int I(\mathbf{n})\zeta_1(n)\,d\Omega_{\mathbf{n}}; \quad \mu_{12} = T\int I(\mathbf{n})\zeta_1(\mathbf{n})\zeta_2(\tilde{\mathbf{n}})\,d\Omega_{\mathbf{n}}, \tag{4}$$

where $I(\vec{n})$ is the intensity of generation of pairs of quanta with directions $\vec{n}$ and $\tilde{\vec{n}}$; $\zeta_1(\vec{n})$ and $\zeta_2(\tilde{\vec{n}})$ are the quantum efficiences of counters 1 and 2 relative to the corresponding quanta. From (3) and (4) we see that

$$\langle n_1 n_2 \rangle - \langle n_1 \rangle \langle n_2 \rangle \sim \langle n_1 \rangle \langle n_2 \rangle \zeta / \langle n \rangle,$$

where $\langle n_1 \rangle \langle n_2 \rangle$ is the number of "random" coincidences and $\langle n \rangle = \sqrt{\langle n_1 \rangle \langle n_2 \rangle}$; therefore the smaller the time of each individual measurement T, and by the same token the smaller $\langle n \rangle$, the easier it is to observe the effect.

3. Let us consider the general description of the statistics of the field of scattered quanta, which holds true for the case of superradiance, too. We assume that the PL process

begins from Gaussian (in particular - vacuum) fluctuations. Then the fluctuations cease to be Gaussian, as a result of the space-time evolution, already in the given pump field regime. To find the correlation functions of higher orders it is then necessary to know not only the correlation $G_{ij} = \langle E^{(-)}(x_i)E^{(+)}(x_j)\rangle$ (as would suffice in the case of a Gaussian process [3]), but also the nonstationary correlation $B_{ij} = \langle E^{(+)}(x_i)E^{(+)}(x_j)\rangle$, which depends on the pump phase. Indeed, under the indicated conditions, the characteristic function (functional) for the field of the scattered quanta will take the form

$$\begin{aligned} \chi_N(\eta_l^*, \eta_l) &= \langle \exp[\textstyle\sum_l E^{(-)}(x_l)\eta_l]\exp[-\sum_l E^{(+)}(x_l)\eta_l^*]\rangle \\ &= \exp\{\textstyle\sum_{ll'}(-G_{ll'}\eta_l\eta_{l'}^* + \frac{1}{2}B_{ll'}^*\eta_l\eta_{l'} + \frac{1}{2}B_{ll'}\eta_l^*\eta_{l'}^*)\}. \end{aligned} \tag{5}$$

From (5) we readily obtain correlation functions of any order, for example,

$$\langle E^{(-)}(1)E^{(-)}(2)E^{(+)}(3)E^{(+)}(4)\rangle = G_{13}G_{24} + G_{14}G_{23} + B_{12}^*B_{34}. \tag{6}$$

Terms of the form B*B, which violate the Gaussian character of the fluctuations, do not vanish apon averaging over the random pump phase (unlike the B_{ij} themselves); the pump amplitude has been assumed determined in this case. These are the terms responsible for the satisfaction of the Manly-Rowe relations and for the correlation of the numbers of quanta of different modes. It is significant that all the correlation functions with equal numbers $E^{(+)}$ and $E^{(-)}$ [1] contain the quantities B_{ij} and B_{ij}^* only in the form of the indicated combination $B_{ij}^*B_{k\ell}$.

4. The foregoing statements concerning the properties of the correlation functions of arbitrary order are valid also in the exact quantum theory in the regime of a given classical (c-number) pump field. If we have in mind normally-ordered correlation functions (see [3]) then, using Mollow's results [4], we con formulat the following prescription for the calculation of $G_{ij}(t)$ and $B_{ij}(t)$ in quantum theory (we assume for simplicity that $B_{ij}(0) = 0$).

Let the field be expanded in the discrete modes. Then the prescription is as follows: a) we add to the function $G_{ij}(0)$ the "vacuum fluctuations" (1/2 photon per mode): $\tilde{G}_{ij}(0) = G_{ij}(0) + \delta_{ij}/2$ (if the generation starts from the vacuum state, then $G_{ij}(0) = 0$); b) starting from the function $\tilde{G}_{ij}(0)$, we solve the classical linear problem of finding $\tilde{G}_{ij}(t)$ and $\tilde{B}_{ij}(t)$; we subtract the"vacuum fluctuations" $\delta_{ij}/2$ from the function $\tilde{G}_{ij}(t)$ and leave $\tilde{B}_{ij}(t)$ unchanged: $G_{ij}(t) = \tilde{G}_{ij}(t) - \delta_{ij}/2$; $B_{ij}(t) = \tilde{B}_{ij}(t)$.

It is easy to show that in the presence of damping in the system it is necessary, in step b, to substitute for the intensity $j(t)$ of the Gaussian noise corresponding to this damping the expression $\langle j^*(t)j(t+\tau)\rangle = \gamma(\nu + \frac{1}{2})\delta(\tau)$, where γ is the damping constant and $\nu = [\exp(\hbar\omega/kT) - 1]^{-1}$ is the average equilibrium number of quanta at the frequency ω and at the temperature T. In optics usually $\nu(\omega) = 0$. The remaining stages of the calculation remain unchanged.

For convenience, we have spoken above of the time evolution of the scattered light. It can be shown [5] that all the conclusions remain in force also for PL that is stationary in

[1] These are precisely the correlation functions necessary for the description of photostatistics experiments.

time but inhomogeneous in space. In this case G(0) should be taken to mean the fluctuations on entering the crystal, and G(t) and B(t) the same on emerging from the crystal. In all other respects, the prescriptions a, b, and c remain in force.

In addition to obtaining more detailed information on the space-time picture of the PL process, the statistics of the photocounts in the PL process can find also the following applications:

1)Inasmuch as the quanta are produced in a small spatial region (within the volume of the crystal), PL serves as a source of pairs of quanta of different frequencies with well-correlated instants of production. By studying the temporal statistics of quantum pairs passing through different devices we are able to measure the difference of group delays in time at two different frequencies.

2) A system of a PL-crystal and counter 1 (see the figure) in one half-space, with quantum efficiency $\zeta_1 = 1$, can be regarded as an example of a system at whose output there are generated (in the direction $\vec{n}$) states with a definite number of quanta. We know of no other proposed system (even hypothetical) for this purpose.

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MAGNETISM OF CONDUCTION ELECTRONS IN THE PRESENCE OF AN ELECTRIC FIELD

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We consider in this paper the magnetism of an electron gas situated in a homogeneous constant magnetic field $\vec{B}$ and in an alternating electric field $\vec{E} = \vec{E}_0 \cos\omega t$, where $\vec{E}_0$ is the amplitude, ω the frequency, and $\vec{E} \perp \vec{B}$.

Let H be the Hamiltonian of the system of electrons with concentration n. Then, in the natural system of coordinate system, the magnetization is equal to

$$M = -\frac{1}{V} \mathrm{Sp}\left(\frac{\partial \hat{H}}{\partial B}\rho\right), \tag{1}$$

where V is the volume of the system and ρ is the density matrix, which satisfies the Neumann equation with Hamiltonian $\hat{H}$. Solving the Neumann equation and substituting ρ in (1) we obtain, after time averaging,

$$M = M_0 - \frac{e^3 n}{2m^2 c} \frac{\omega_c}{(\omega_c^2 - \omega^2)^2} E_0^2, \tag{2}$$