EFFICIENCY OF GENERATION AND POSSIBLE OBSERVATION OF GRAVITATIONAL RADIATION

Peter J. Westervelt
Physics Dept., Brown University, Providence, R. I. [1)]
ZhETF Pis'ma 4, No. 9, 333-338, 1 November 1966

This letter discusses the practical realizability of recently proposed [1,2] experiments on the generation and observation of gravitational waves [2)]. We use the classical theory of gravitational bremsstrahlung [3,4] to estimate the upper limit of efficiency of the generation of gravitational energy.

Using the notation of Landau and Lifshitz [5], the energy flux in a plane weak gravitational wave propagating in the x direction can be expressed by

$$ct^{01} = c^3(16\pi k)^{-1}[\dot{\psi}_{23}^2 + \tfrac{1}{4}(\dot{\psi}_{22} - \dot{\psi}_{33})^2], \tag{1}$$

or

$$ct^{01} = c^3(64\pi k)^{-1}[\dot{\psi}^2 + 4(\dot{\psi}_{23}^2 - \dot{\psi}_{22}\dot{\psi}_{33})]. \tag{2}$$

Since $\psi = \psi_{22} + \psi_{33}$, these expressions are identical. For cylindrically-symmetrical radiators and in a suitable coordinate frame [3] ψ_{23} and one of either ψ_{22} or ψ_{33} vanish, and (2) takes the form

$$ct^{01} = c^3(64\pi k)^{-1}\dot{\psi}^2, \tag{3}$$

yielding for the gravitational Poynting vector

$$S = -(64\pi k)^{-1}c^4\dot{\psi}\nabla\psi\ . \tag{4}$$

This result simplifies the treatment of certain cylindrically symmetrical fields, since ψ satisfies the scalar wave equation

$$\Box\psi = -16\pi c^{-4}k\tau, \tag{5}$$

in which $\tau = -\mu_0 c^2$ for nonrelativistic macroscopic bodies, with μ_0 the time-dependent rest-mass density.

In an earlier paper [4] the author obtained the cylindrically-symmetrical components of the gravitational waves resulting from emission and absorption of a light pulse of infinitesimally short duration. Using these results, we can readily show that $\dot{\psi}$ is the sum of monopole and dipole fields produced by the radiator and the absorber, so that

$$\dot{\psi} = [\dot{\psi}_m + \dot{\psi}_d]_{em} + [\dot{\psi}_m + \dot{\psi}_d]_{abs}.$$

In the wave zone

$$\dot{\psi} = 4c^{-3}kpR_0^{-1}[(1 - \cos\varphi)^{-1}\sin^2\varphi]\{\delta[x^0 - R_0] - \delta[x^0 - R_0 - L(1 - \cos\varphi)]\}, \tag{6}$$

where R_0 is the length of the vector from the radiator to the field point, L the length of the vector from the radiator to the absorber, and φ the angle between R_0 and L. The field $\dot{\Psi}(t)$ resulting from the electromagnetic wave with arbitrary momentum F(t) can be obtained from the Green's function, namely

$$\dot{\Psi}(t) = p^{-1}\int_{-\infty}^{+\infty} \dot{\psi}(t - t')F(t')dt'. \tag{7}$$

For a rectangular wave pulse with finite duration T and total momentum p we have

$$F(t') = pT^{-1}[\theta(t') - \theta(t' - T)]. \tag{8}$$

Combining (7) and (8) we obtain for the field $\dot{\Psi}$

$$\begin{aligned}\dot{\Psi} = {} & 4c^{-3}kpT^{-1}R_0^{-1}[(1 - \cos\varphi)^{-1}\sin^2\varphi]\{\theta[ct - R_0] - \theta[ct - R_0 - cT] \\ & - \theta[ct - R_0 - l(1 - \cos\varphi)] + \theta[ct - R_0 - L(1 - \cos\varphi) - cT]\}.\end{aligned} \tag{9}$$

The total radiation energy is obtained from this equation by integrating (3) or (4) over all of time and over a surface surrounding the source. For short pulses, when $T \leq 2L/c$, we have

$$\epsilon = (8/3)k(cp)^2c^{-5}T^{-1}[1 - (3/4)(cTL^{-1}) + (1/4)(cTL^{-1})^2 - (1/32)(cTL^{-1})^3], \tag{10}$$

and for long pulses, when $T \geq 2L/c$,

$$\epsilon = (4/3)k(cp)^2Lc^{-6}T^{-2}. \tag{11}$$

Before we discuss this result, it is of interest to calculate the flux of gravitational energy and the total power resulting from the emission and absorption of an ideally collimated light beam when the beam intensity is 100% modulated at a frequency ω, i.e.,

$$F(t') = c^{-1}P_E(1 - \cos\omega t'), \tag{12}$$

where P_E is the average energy of the light beam. Combining (12) and (7) we can find the gravitational field, which we can insert in (4) and obtain, after averaging over the time,

$$S = (kc^{-5})P_E^2(4\pi R_0^2)^{-1}\{1 - \cos[k_m L(1 - \cos\varphi)]\}[(1 - \cos\varphi)^{-2}\sin^4\varphi]n, \tag{13}$$

where $n = \vec{R}_0/R_0$, $k_m = \omega/c$, and L is the distance between the radiator and absorber. The total gravitational power P_G, obtained by integrating (13) over the closed surface, is:

$$P_G = (4/3)(kc^{-5})P_E^2[1 + (3/4)(k_m L)^{-3}(\sin 2k_m L - 2k_m L)]. \tag{14}$$

When the distance between radiator and absorber is large compared with the gravitational wavelength, $k_m L \gg 1$ and

$$P_G = (4/3)(kc^{-5})P_E^2, \tag{15}$$

the gravitational power does not depend on the modulation frequency [3]. The results contained in (13) and (15) agree with those of Rose [3].

An upper bound for the gravitational power can be obtained from (15). The ratio P_G/P_E is equal to the beam power divided by $(3/4)c^5k^{-1}$ - an astronomically large constant $\sim 10^{60}$ erg/sec. This cosmic constant is by accident equal to the total power crossing the optical edge of the expanding Friedmann universe; it is also equal to the rate of generation of the rest energy in Hoyle's stationary model. We thus find that no phenomenon occurring on a planetary, solar, or galactic scale can generate measurable gravitational power [4]. We exclude here, of course, the power transferred by gravitational induction fields, such as tide effects.

Finally, the efficiency with which short pulses of gravitational energy are generated can be obtained from (10) with $T \ll 2L/c$, namely

$$\epsilon_G = (8/3)k(pc)^2c^{-5}T^{-1}. \tag{16}$$

The energy produced by radiation (neglecting absorption) is $\epsilon_G/2$, and putting for the light pulse $pc = \epsilon_E$, we get

$$(1/2)\epsilon_G/\epsilon_E = (4/3)(kc^{-5})\epsilon_E/T. \tag{17}$$

This quantity is again negligibly small for all real values of the energy ϵ_E or pulse duration T.

The author is grateful to R. Siemann [6] and G. Ludwig [7] for checking the applicability of the scalar theory. P. Hock [8] made the preliminary calculation of the gravitational-pulse radiation. S. Morin [9] made the preliminary calculations of the absorption of the energy by an active detector in an external gravitational field; these calculations were refined by F. Cooperstock [10].

[1] U. Kh. Kopvillem and V. R. Nagibarov, JETP Letters 2, 529 (1965), transl. p. 329.
[2] V. B. Braginskii, UFN 86, 433 (1965), Soviet Phys. Uspekhi 8, 513 (1966).
[3] M. E. Rose, Ed., Proceedings of the Eastern Theoretical Physics Conference, Gordon and Breach, Science Publishers Inc., N. Y. - London, 1963.
[4] P. J. Westervelt, Acta Physica Polonica 27, 831 (1965).
[5] L. D. Landau and E. M. Lifshitz, Classical Theory of Fields, Addison-Wesley Co., Inc. Reading, Mass., USA, 1962.
[6] R. Siemann, Bachelor of Science Thesis, Department of Physics, Brown University, 1964.
[7] G. Ludwig, Doctor of Philosophy Thesis, Department of Physics, Brown University, 1966.
[8] P. Hoch, Master of Science Thesis, Department of Physics, Brown University, 1964.
[9] S. Morin, Bachelor of Science Thesis, Department of Physics, Brown University, 1965.
[10] F. Cooperstock, Doctor of Philosophy Thesis, Department of Physics, Brown University, 1966.

1) Work supported in part by the USA Atomic Energy Commission.

2) The high-frequency gravitational radiation, which the authors of [1] claim to result from oscillations of the quadrupole moment and electron cloud, cannot occur because this moment is orthogonal to the charge moment in the following sense: The distribution of the

quadrupole charge $D^e_{\alpha\beta}$ over the surface can radiate a highly collimated beam of electromagnetic waves in a direction perpendicular to the surface. A quadrupole of mass $D^m_{\alpha\beta}$, proportional to $D^e_{\alpha\beta}$, will not emit the corresponding gravitational power, since the angular distribution of the intensity of the gravitational radiation

$$dI^m = k(36\pi c^5)^{-1}[\tfrac{1}{4}(\dddot{D}^m_{\alpha\beta}n_\alpha n_\beta)^2 + \tfrac{1}{2}(\dddot{D}^m_{\alpha\beta})^2 - \dddot{D}^m_{\alpha\beta}\dddot{D}^m_{\alpha\gamma}n_\beta n_\gamma]d\Omega$$

differs radically from the angular distribution of the electromagnetic intensity, given by

$$dI^e = (36\pi c^5)^{-1}[\tfrac{1}{4}\dddot{D}^e_{\alpha\beta}\dddot{D}^e_{\alpha\gamma}n_\beta n_\gamma - \tfrac{1}{4}(\dddot{D}^e_{\alpha\beta}n_\alpha n_\beta)^2]d\Omega.$$

In particular, dI^m vanishes in the direction in which dI^e has a maximum.

[3] In the case when $k_m L \ll 1$, the radiator becomes a pointlike quadrupole, and (14) yields a result identical with the time-averaged $1/45\ (kc^{-5})\dddot{D}^2_{\alpha\beta}$, where $D_{\alpha\beta}$ is the tensor of the quadrupole moment of the system, including the energy of the electromagnetic field.

[4] The author does not believe that passive detectors of gravitational waves can be realized in principle. Active detectors, i.e., generators of gravitational waves, can measure, in principle, the radiation from another source. This question will be dealt with in a future paper by the author (with F. Cooperstock and G. Ludwig).

EFFECT OF UNIAXIAL COMPRESSION ON THE PARAMAGNETIC RESONANCE OF Nd^{3+} IN $CaWO_4$

M. M. Zaitov and L. Ya. Shekun
Kazan' State University
Submitted 18 July 1966
ZhETF Pis'ma 4, No. 9, 338-341, 1 November 1966

An investigation of the spectra in a deformed crystal is the most direct method of determining the interaction between an impurity ion and its environment [1,2]. The effect of pressure on EPR spectra was investigated so far for iron-group ions [3]. We observed the influence of uniaxial pressure on the EPR of certain rare-earth ions in single-crystal scheelites. The present communication is devoted to Nd^{3+} in $CaWO_4$.

The main term $4f^3\ {}^4I_{9/2}$ of neodymium in the $CaWO_4$ lattice splits into five Kramers doublets spaced ~100 cm^{-1} apart. At 4.2°K, the line from the lower doublet is observed, with $g_\parallel = 2.03$ and $g_\perp = 2.54$ [4] (we do not consider the hyperfine structure). Pressure applied to the crystal adds to the ordinary spin-Hamiltonian with effective spin 1/2 a perturbation $\mathcal{H}'$ linear in the magnetic field and in the spin [5]:

$$\mathcal{H}' = G_{ik\ell m}S_i H_k u_{\ell m}. \tag{1}$$

Here i, k, ℓ, m = x, y, z; $u_{\ell m} = \frac{1}{2}(\partial u_\ell/\partial x_m + \partial u_m/\partial x_\ell)$ is the strain tensor. The tensor G determines the magnitude of the effect. It is symmetrical in the second pair of indices, and its remaining properties are determined by the symmetry of the local field. In this case the symmetry is S_4 [6], but we can assume D_{2d} with good approximation [7]. Then G has