$$G_{31} \approx \frac{s_{13}}{s_{11} + s_{12}} G_{33}. \quad (7)$$

The elastic tensor s, defined by

$$-u_{ik} = s_{iklm}\sigma_{lm} \quad (8)$$

for $CaWO_4$, was not measured, but the results of [8] show that $s \sim 10^{-12}$ cm^2/dyne, yielding the estimate

$$G \sim 10^{-20} \text{ erg/G}. \quad (9)$$

Even this preliminary estimate leads to a few interesting conclusions, particularly to a comparison of the Kramers doublets belonging to the ions of the iron group and of rare earths. Tucker [9] made measurements on the Kramers doublet of the ion Co^{2+} ($3d^7$) in MgO and obtained (in our notation) $G \sim 10^{-19}$ erg/G. Thus, the direct coupling of the Nd^{3+} ion to the $CaWO_4$ lattice turns out to be unexpectedly small and, most curiously, smaller than for Co^{2+} in MgO. This contradicts the current opinion that rare-earth ions are more strongly coupled dynamically to the lattice than the iron-group ions.

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SELF-FOCUSING OF POWERFUL LIGHT BEAMS BY THERMAL EFFECTS

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Submitted 18 July 1966
ZhETF Pis'ma 4, No. 9, 341-345, 1 November 1966

Self-focusing [1-3] of powerful light beams is usually associated with the Kerr effect and electrostriction [3-5], since these are the very mechanisms that lead to the strongest dependence of the dielectric constant of a substance on the field amplitude. The purpose of this communication is to call attention to a possible self-focusing of light by heating of the medium in the field of the wave [1)].

The dielectric constant of a medium, taking heating into account, can be represented

by [2]

$$\epsilon(T) = \epsilon_0(T_0) + \frac{\partial \epsilon}{\partial T} T', \tag{1}$$

where T_0 is the unperturbed temperature of the medium and T' the perturbation of the temperature in the field of the wave. The necessary condition for self-focusing is the requirement that the dielectric constant of the medium increase in the field region, i.e., the condition $\partial\epsilon/\partial T > 0$. For most substances $\partial\epsilon/\partial T < 0$, and heating is accompanied by defocusing of the beam. At the same time, many substances, such as calcite, sapphire, fused quartz, water, etc., have a positive derivative $\partial\epsilon/\partial T$ in a definite temperature and wavelength interval.

To determine T' we use the heat-conduction equation

$$\rho c_p \frac{\partial T'}{\partial t} = Q + \kappa \Delta T', \tag{2}$$

where κ is the coefficient of thermal conductivity of the medium, ρc_p the specific heat per unit volume, Q the distribution of the heat sources, which has the form

$$Q = \alpha |E|^2 c/8\pi,$$

α is the coefficient of linear attenuation of the light in the medium, c the velocity of light, and E the electric-field amplitude.

Since heat-conduction processes do not have time to affect the temperature distribution within a time of the order of the duration of the laser pulse, $\tau_p \sim 10^{-8} - 10^{-3}$ sec [3], the perturbation T' is a quasilocal function of the electric field amplitude E. To estimate the effect, let us consider the case of a field of constant amplitude E_0 applied suddenly at a given point, when expression (1) for the dielectric constant can be rewritten in the standard form for an isotropic cubic medium

$$\epsilon = \epsilon_0 + \epsilon_T' E_0^2.$$

Here ϵ_T' is a linearly increasing function of the time:

$$\epsilon_T' = \frac{1}{8\pi} \frac{\partial \epsilon}{\partial T} \frac{\alpha c t}{\rho c_p}$$

$\rho = 2$ g/cm^2, $c_p = 0.5$ J/g-deg, $\partial\epsilon/\partial T = 10^{-4}$ deg^{-1}, $\alpha = 10^{-2}$ cm^{-1}, and $\epsilon_T' = 2 \times 10^{-13} t_{(\text{nsec})}$ is in cgs esu, with $t_{(\text{nsec})}$ the time reckoned from the instant of arrival of the light pulse. For realistic pulse durations $\tau_p \sim 20$ nsec, the coefficient is $\epsilon_T' \simeq 4 \times 10^{-12} (t/\tau_p)$ cgs esu, which for $t \simeq \tau_p$ is of the same order of magnitude as the coefficient of nonlinearity of liquids due to the Kerr effect ($\epsilon_{K\ \max}' \simeq 10^{-11}$ cgs esu).

An important feature of nonstationary thermal effects is their integral character - the perturbation of the dielectric constant depends not on the pulse power, as in the Kerr effect, but on its energy, i.e., the light beam is focused because the preceding part of the light pulse has acted on the medium. For example, the change of the dielectric constant of the medium on the beam axis (r = 0), resulting from the passage of a light pulse in a medium having the parameters listed above, is of the order of magnitude $\partial\epsilon \simeq 4 \times 10^{-5} W_{(J)}/a^2_{(mm)}$,

where $W_{(J)}$ is the total pulse energy in Joules and $a_{(mm)}$ the width of the beam in millimeters. This gives grounds for hopting to observe self-focusing of laser beams operating in the spike mode.

When a cw laser is used, a stationary ($\partial/\partial t = 0$) temperature distribution is established in the transverse section of the beam after a time $t \gg a^2/4\kappa$, this distribution being always broader than the corresponding field distribution, i.e., the temperature is a nonlocal function of the field E. As a result, the usual picture of self-focusing of the beam is greatly altered. In particular, for a beam with Gaussian form $E = E_0 \exp(-r^2/2a^2)$ the distribution of the refractive index can be approximated by the parabola [4] $n = n_0 - n_2 r^2/2$, where the coefficient n_2 depends on the parameters of the medium and of the beam:

$$n_2 = \beta/a^2, \quad \beta = \frac{\pi(\partial\epsilon/\partial T)P\alpha}{\kappa n_0^2}; \tag{3}$$

P is the beam power. As is well known, the Gaussian form of the beam is not altered by propagation in such a medium, and only the width changes, $a = a(z)$. The equation for the dimensionless "half-width" of the beam $a_H = k_0 a$, where $k_0 = \omega\sqrt{\epsilon_0}/c$, takes in this case the form

$$\frac{d^2a}{dz^2} = a^{-1}(a^{-2} - \beta). \tag{4}$$

It follows from (4) that if $\beta \gg 1/a_0^2$ and $\dot{a} = 0$ on entering the nonlinear medium, the beam becomes self-focused, and in the geometrical-optics approximation the focusing length is $L_f = \sqrt{\pi/2}(a_0/\sqrt{\beta})$ [5]. With decreasing beam width, the refractive term is cancelled out by the diffractive one, and with further beam propagation its width oscillates about a certain average width $\tilde{a} = 1/\sqrt{\beta}$. For $\kappa = 0.1$ J/cm-sec, $\partial\epsilon/\partial T = 10^{-4}$ deg, $\alpha = 10^{-2}$ cm^{-1}, and $n_0 = 2$ we get $\beta \simeq 10^{-5}P$ (watts).

It is of interest both to check experimentally the foregoing estimates and to ascertain in what class of substances is self-focusing of powerful light beams by thermal effects possible.

In conclusion we note that thermal effects can change radically the picture of ordinary self-focusing in nonlinear liquids with $\partial\epsilon/\partial T < 0$.

The author is grateful to V. I. Talanov for valuable advice and a discussion of the results.

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1) The question of thermal influence on the character of propagation of light beams in media is not new. It is well known, for example, that heating of the laser medium in the pump field leads to a marked change in the lasing conditions [6].

2) For simplicity we confine ourselves to the case when heating does not give rise to anisotropy of the medium.

3) The time to equalize the temperature over a dimension a by thermal conductivity is $t_\kappa = a^2/4\kappa$; even in good heat conductors we have $t_\kappa > 10^{-1}$ sec for $a = 10^{-1}$ cm.

4) In the paraxial region of the beam $r \ll a$, the parabolic relief is an exact solution of the stationary heat conduction equation with Gaussian source; we do not present the exact solution of the equation for arbitrary r.

5) The results are valid provided the self-focusing length is much shorter than the beam attenuation length, $L_f \ll 1/\alpha$, i.e., provided $\alpha \ll (2\partial\epsilon/\partial T)P/a_0^2\kappa n_0$.

MASS-SPECTROMETRIC OBSERVATION OF LONG-LIVED AUTO-IONIZATION STATES OF THE IONS Ca^+ AND Sr^+

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Submitted 19 July 1966
ZhETF Pis'ma 4, No. 9, 345-348, 1 November 1966

Auto-ionization states of the lithium-like ions N^{4+} and O^{5+}, which remain excited for $\sim 5 \times 10^{-8}$ sec, were recently observed [1]. The lifetimes [2] of lithium, potassium, and rubidium atoms in the auto-ionization state is much larger, $\sim 5 \times 10^{-6}$, 9×10^{-5}, and 7.5×10^{-5} sec, respectively. It is known from other investigations [3,4] that rapid auto-ionization processes are produced by collisions of electrons with calcium atoms.

From an analysis of all these investigations one would expect that Ca^+ and Sr^+ ions could be produced in auto-ionization states that retain their excitation long enough ($>10^{-6}$ sec) to render them observable with the aid of a mass spectrometer. In other words, we expect that following collision of electrons with Ca and Sr atoms, the Ca^+ and Sr^+ ions, which are isoelectronic to K and Rb, respectively, can attain excited states with lifetimes, relative to either photon or electron emission, longer than the characteristic times of these processes for non-forbidden transitions. The purpose of this paper is to prove the foregoing.

The investigation was carried out with a double mass spectrometer [5]. A beam of Ca^+ or Sr^+ ions accelerated to 2.8 keV was separated by the first magnetic analyzer and guided through a slit into a chamber located between the first and second magnetic analyzers. The doubly charged Ca^{2+} or Sr^{2+} ions produced in the chamber were separated from the beam of the initial singly-charged ions by the second magnetic analyzer and recorded with electrometric amplifiers. These doubly-charged ions could be formed from the singly-charged ions (A^+) via