Fig. 2, which shows also for comparison the spectrum of mercury and the green line of the second harmonic of the neodymium laser ($\lambda = 0.529\ \mu$).

Generation at the difference frequency ($\lambda_4 = 2.02\ \mu$) is also of definite interest; such a subtraction of the neodymium laser frequency from the ruby laser frequency can be realized in a nonlinear $LiNbO_3$ crystal. The calculated synchronism angle (see [17]) for the interaction $k_1 - k_2 = k_4$ is $\theta_s = 50°40'$. It must be noted that owing to the Manley-Rowe relation the maximum coefficient of energy-into-radiation transformation at the difference frequency does not exceed $\eta = \omega_4/\omega_1 = 30\%$.

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[1] S. A. Akhmanov and R. V. Khokhlov, Problemy nelineinoi optiki (Problems of Nonlinear Optics), VINITI, 1964.
[2] P. A. Franken and I. F. Word, Rev. of Mod. Phys. 35, 23 (1963).
[3] M. Bass, P. A. Franken, A. E. Hill, C. W. Peters, and G. Weinreich, Phys. Rev. Lett. 8, 18 (1962).
[4] I. Giordmaine, ibid. 8, 19 (1962).
[5] R. Miller and A. Savage, Phys. Rev. 128, 2175 (1962).
[6] A. Smith and N. Braslau, I.B.M. Res. and Developm. 6, 361 (1962).
[7] D. H. McMahon and A. R. Franklin, J. Appl. Phys. 36, 2073 (1965).
[8] P. N. Butcher, W. H. Kleiner, P. L. Kelley, and H. I. Zeiger, Program of Phys. of Quantum Electronics Conf. June, 1965, Puerto-Rico, p. 13.
[9] I. Giordmaine, ibid. p. 13.
[10] M. D. Martin, E. L. Thomas, and I. K. Wright, Phys. Lett. 15, 136 (1965).
[11] T. Yajima and M. Takatsuji, Z. Angew. Mathemat. und Phys. 16, 34 (1965).
[12] T. Yajima and M. Takatsuji, J. Phys. Soc. Japan 19, 2343 (1964).
[13] N. Tran, I. Spalter, I. Hanus, I. Ernest, and D. Kehl, Phys. Lett. 19, 285 (1965).
[14] N. Tran and D. Kehl, Appl. Optics 5, 168 (1966).
[15] D. A. Kleinman, Phys. Rev. 126, 1977 (1962).
[16] Fr. Zernike, J. Opt. Soc. of America 54, 1215 (1964).
[17] C. D. Boyd, R. C. Miller, K. Nassan, W. L. Bond, and A. Savage, Appl. Phys. Lett. 5, No. 11 (1964).

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# SINGULARITIES OF THE FARADAY EFFECT IN n-InSb IN THE MILLIMETER BAND

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We investigated the Faraday effect in n-type InSb at 77.8°K as a function of the magnetic field $\vec{B}$ and of the sample thickness.

As is well known, in a magnetized semiconductor in which an electromagnetic wave propagates in the direction of a constant magnetic field the natural modes have circular polarization [1,2]. The propagation constants of the waves polarized in the right and left directions are given by

$$\alpha_{\pm} = (\omega/c\sqrt{2})[(\epsilon_{\pm}'^{2} + \epsilon_{\pm}''^{2})^{1/2} + \epsilon_{\pm}']^{1/2} \tag{1}$$

where $\omega$ is the frequency of the electromagnetic wave, c the velocity of light,

$$\epsilon_{\pm}' = \epsilon_{st} - p\{(1 \pm q)/[s^{2} + (1 \pm q)^{2}]\}, \quad \epsilon_{\pm}'' = ps/[s^{2} + (1 \pm q)^{2}],$$

$$q = \omega_{c}/\omega, \quad p = \omega_{p}^{2}/\omega^{2},$$

$\omega_c$ the cyclotron frequency, $\omega_p$ the plasma frequency, $\tau$ the effective free-path time 1), $\epsilon_{st} = 16.6$ the dielectric constant of the InSb lattice, and the signs + and - pertain to the left-hand and right-hand wave polarizations. It follows from (1) that the polarization plane of a linearly-polarized wave is rotated after traversing a path $l$ in the semiconductor through an angle

$$\theta = \tfrac{1}{2}(\alpha_{-} - \alpha_{+})l. \tag{2}$$

Taking into account the reflections of the electromagnetic wave from the boundary surfaces, the expression for $\theta$ becomes [3]:

$$\theta = \tfrac{1}{2}(\alpha_{-} - \alpha_{+}) + \tfrac{1}{2}\xi, \tag{3}$$

where

$$\tan\xi = \frac{K_{+}'K_{-}'' - K_{-}'K_{+}''}{K_{+}'K_{-}' + K_{+}''K_{-}''}, \quad K_{\pm} = K_{\pm}' + iK_{\pm}'' = \frac{1 - R_{\pm}}{1 - R_{\pm}\exp(-2\beta_{\pm}l - i2\alpha_{\pm}l)}.$$

Here

$$R_{\pm} = \left(\frac{\beta_{\pm} + i\alpha_{\pm} - i\alpha_{0}}{\beta_{\pm} + i\alpha_{\pm} + i\alpha_{0}}\right)^{2}$$

is the coefficient of reflection from one face, $\alpha_0 = \omega/c$, and

$$\beta_{\pm} = (\omega/c\sqrt{2})[(\epsilon_{\pm}'^{2} + \epsilon_{\pm}''^{2})^{1/2} - \epsilon_{\pm}']^{1/2}$$

is the damping constant. It follows from (3) that the Faraday angle is an oscillating function of the layer thickness $l$.

The experimental setup included a klystron oscillator operating at $\lambda \simeq 4$ mm, attenuators, a measuring head, and an indicator showing the power passing through the sample. The measuring head consisted of a stationary rotary wave transformer converting the $H_{10}$ mode in a rectangular waveguide into an $H_{11}$ mode in a round waveguide (3 mm dia.), and a sample holder permitting power to flow through a cylindrical sample (3 - 2 mm dia.) or through a sample in the form of a parallelepiped. The rotary transformer was coupled to a dial to read the angle. The position of the polarization plane was indicated by the minimum of the detector reading. The measuring head with the sample was placed in liquid nitrogen. The axial magnetic field was produced by a solenoid. The investigated sample was cut from an ingot of n-type InSb compensated with germanium, having nominal parameters $n = 5.69 \times 10^{13}$ cm$^{-3}$ and $\mu = 2.96 \times 10^5$ cm$^2$/V-sec. The theoretical curves were calculated from (2) and (3) for $n = 5.69 \times 10^{13}$ cm$^{-3}$ and $\mu = 3.9 \times 10^5$ cm$^2$/V-sec, which is within the deviations from the nominal values due to the inhomogeneity of the ingot.

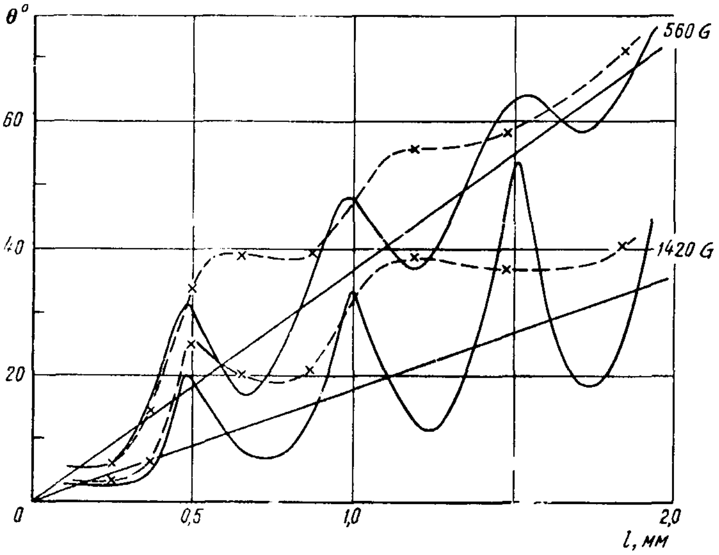

Fig. 1. Angle of rotation of the polarization plane vs. semiconductor sample thickness in magnetic fields above cyclotron resonance (the straight lines were calculated from (2), the continuous curves from (3), and the dashed ones are experimental).

Figure 1 shows the rotation of the polarization plane against the sample thickness at magnetic fields stronger than the cyclotron-resonance field. The strong oscillatory deviation of the experimental curves and of those calculated from (3) from the corresponding straight line given by (2) shows the strong influence of reflections from the boundary planes. At sample thicknesses that are multiples of the length of the electromagnetic wave, "geometric" resonance takes place in the sample, and the Faraday angle is maximal in this case. With increasing magnetic fields, the peaks of the oscillations become sharper, this being attributed to the decreased losses in the semiconductor.

Plots of the Faraday angle vs. the magnetic field are shown in Fig. 2. A distinguishing feature of these curves is the negative direction of the rotation angle in weak magnetic fields. According to (2) this occurs for samples with mobility $\mu > e/\omega m^*$, meaning in our case $\mu > 2.98 \times 10^5$ cm$^2$/V-sec. The experimental and calculated curves agree well qualitatively. The quantitative differences are due to the approximate nature of the theory, in which no account is taken of the energy dependence of $\tau$, and which is valid only for waves in free space, whereas in the experiment the actual propagation was in a waveguide of 3 mm diameter filled with a semiconductor.

It should be noted in conclusion that the large Faraday angles and the relatively small damping in strong magnetic fields make it possible to employ this phenomenon in practice in non-reciprocal microwave devices such as ferrites.

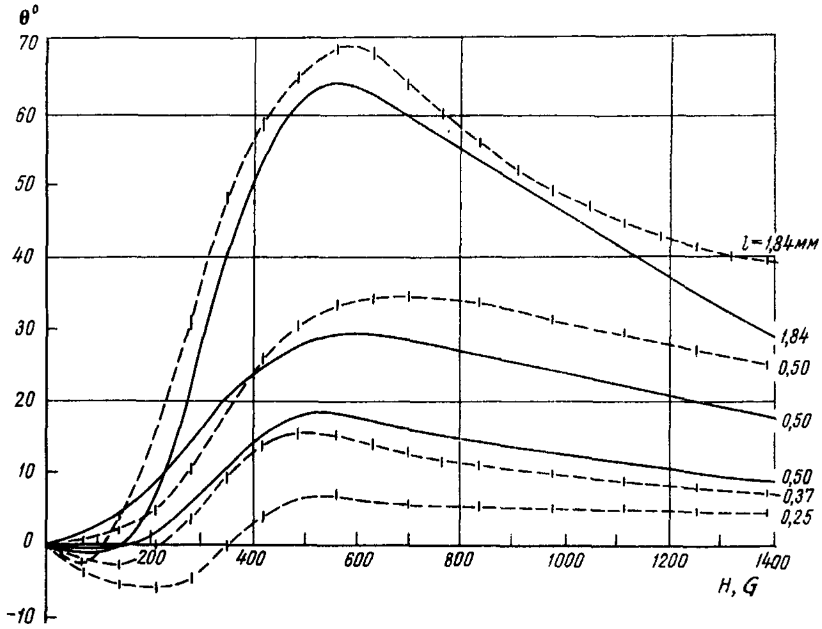

Fig. 2. Rotation of the polarization plane vs. magnetic field (curves I and II calculated from (3), curve III from (2), and the dashed curves are experimental).

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[1] I. K. Furdyna and S. Broesma, Phys. Rev. 120, 1996 (1960).

[2] T. Moss. Optical Properties of Semiconductors (Russ. Transl.), IIL, 1961.

[3] B. Donovan and T. Medcalf, Brit. J. Appl. Phys. 15, 1139 (1964).

[4] V. L. Ginzburg, Rasprostranenie elektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in a Plasma), Fizmatgiz, 1960.

1) If $\tau$ depends on the energy, the accuracy of expressions (1) - (3) is the higher, the stronger the inequality $\omega\tau > 1$ [4].

In the article "Singularities of the Faraday Effect in n-InSb in the Millimeter Band" by V. M. Afinogenov et al. (Vol 4, No. 11, p. 300), add to the unnumbered equation following Eq. (1) the definition $s = 1/\omega\tau$. On p. 301, line 3 from the top replace $n = 5.69 \times 10^{13}$ cm$^{-3}$ with $n = 5.69 \times 10^{12}$ cm$^{-3}$. The curves of Fig. 2 on p. 302 should be numbered I, II, and III reading from top to bottom.