sary to turn to the experimental data.

In principle, the nuclear shape always changes jumpwise when the number of nucleons changes. But this is due simply to the discreteness of nuclear matter, by virtue of which both the deformation energy E_{def} and the pairing energy E_{pair} always change by finite amounts. For a certain number of nucleons the difference $E_{def} - E_{pair}$ should reverse sign and change numerically by a finite amount, and this corresponds to a jumplike change in shape.

We note that in many cases the existence of relatively small deformations can be regarded as reliably established. Thus for the even-even nucleus Os^{190}, which has a well pronounced rotational level structure, the deformation parameter is $\delta = 0.15$. The deformations of the odd nuclei Ir^{191} and Ir^{193} are 0.14 and 0.12 respectively. In our opinion the deformations of the odd isotope of gold and of the odd isotopes of europium and promethium, which pertain to a different transition region, may be even smaller [6,7].

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FACTORIZATION OF AMPLITUDES AND PAIR PRODUCTION OF RESONANCES AT HIGH ENERGIES

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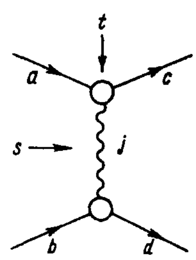
Particle collisions at high energies can now be described by means of several models in which amplitude factorization is possible. The reaction amplitudes are described by diagrams of the type shown in the figure, where j is the spin of the virtual particle and can be either integer (in the case of the peripheral model) or complex (in the case of Regge poles). We consider in this note a possible check on amplitude factorization in pair production of resonances with nonzero spins at high energies.



Let us consider the reaction $a + b \to c + d$ in which the masses and spins of the participating particles are arbitrary. The final state of the system of particles c and d is described by a polarization density matrix $\rho^{(f)mm'}_{nn'}$, where m, m' and n, n' are the spin indices of particles c and d respectively. Let the spin states of the initial particles be uncorrelated, i.e., let the polarization density matrix of the initial particles be

$$\rho^{(i)kk'}_{\ell\ell'} = \rho^{kk'}_{a} \; \rho^{B}_{\ell\ell'}.$$

This condition is always satisfied in experiments with elementary particles.

We write the expansion of the helicity amplitudes in the c.m.s. of the t-channel in terms of the states with definite momentum:

$$F^{(t)}_{\lambda_a\lambda_{\bar c}\lambda_{\bar b}\lambda_d}(s, t) = \frac{1}{4\pi} \sum_j (2j + 1) \, f^{j}_{\lambda_a\lambda_{\bar c}\lambda_{\bar b}\lambda_d}(t) \, d^{j}_{\lambda,\mu}(Z_t); \quad \lambda = \lambda_a - \lambda_{\bar c}, \quad \mu = \lambda_{\bar b} - \lambda_d. \tag{1}$$

Using the method developed by Marinov [1], we find that the helicity amplitudes in the c.m.s. of the s-channel are:

$$G^{(s)}_{\lambda_a\lambda_b\lambda_c\lambda_d}(s, t) = G^{(1)}_{\lambda_a\lambda_c}(s, t) \, G^{(2)}_{\lambda_b\lambda_d}(s, t). \tag{2}$$

In deriving (2) we have used in fact not only the condition for factorization of the amplitudes in the t-channel [2]

$$f^{j}_{\lambda_a\lambda_{\bar c},\lambda_{\bar b}\lambda_d}(t) = f^{(1)j}_{\lambda_a\lambda_{\bar c}}(t) f^{(2)j}_{\lambda_{\bar b}\lambda_d}(t), \tag{3}$$

but also the asymptotic behavior of $d^{j}_{\lambda,\mu}(Z_t)$ as $Z \to \infty$

$$d^{j}_{\lambda,\mu}(Z_t) = i^{\lambda-\mu}(Z_t/2)^{j}\Gamma(2j + 1)[\Gamma(j + \lambda + 1)\Gamma(j - \lambda + 1)\Gamma(j - \mu + 1)]^{-1/2}; \qquad Z_t \to \infty, \quad \operatorname{Re} j > -\tfrac{1}{2}. \tag{4}$$

When $s \gg M_i^2$ the condition $Z_t \gg 1$ reduces to the condition

$$|t| \gg [(M_a^2 - M_c^2)(M_b^2 - M_d^2)]/s \qquad \text{or} \qquad |t| \gg [M^2(M_b^2 - M_d^2)^2]/s^2,$$

if $M_a = M_c = M$.

Using (2), we obtain

$$\rho^{(f)mm'}_{nn'} = \frac{\sum_{kk'\ell\ell'} G^{(s)}_{k\ell,mn} \, \rho^{(i)kk'}_{\ell\ell'} \, G^{*(s)}_{k'\ell',m'n'}}{\sum_{kk'\ell\ell'mn} G^{(s)}_{k\ell,mn} \, \rho^{(i)kk'}_{\ell\ell'} \, G^{*(s)}_{k'\ell',mn}} = \rho^{mm'}_{c} \, \rho^{d}_{nn'}, \tag{5}$$

with ρ_c dependent only on ρ_a, and ρ^d on ρ^b.

The factorization condition thus leads to the absence of correlation between the spin states of the produced particles.

In some particular cases, when the particles have large spins, condition (5) follows only from the assumption that the extreme-right singularity (or series of singularities) in the j-plane has definite quantum numbers, viz., signature P_j, parity P, G-parity, and isospin T. Such cases include:

a) $\pi + N \to V + N$, where V = vector meson. Here condition (5) is satisfied for arbitrary possible quantum numbers of the principal singularities.

b) $\pi + N \to V + N^*$, where N^* = arbitrary isobar. For this reaction condition (5) is satisfied only if the quantum numbers of the principal singularities satisfy the relations $PP_j = +1$ and $G(-1)^T P_j = +1$ (vacuum, ω, φ, ρ, and R trajectories).

c) $N + N \to N + N$. Condition (5) is satisfied if the extreme-right singularities have quantum numbers $PP_j = -1$ and $G(-1)^T P_j = +1$ or $PP_j = -1$ and $G(-1)^T P_j = -1$.

An experimental verification of condition (5) is very difficult if c and d are stable particles. It becomes much simpler, however, if c and d are resonances. In this case condition (5) can be verified by studying the correlations between the angular distributions of the decay products of the produced resonances.

Let us consider the joint angular distribution $W(\theta_1, \varphi_1, \theta_2, \varphi_2)$ of the decay products of the resonances c and d. Here θ_1, φ_1 and θ_2, φ_2 are the polar and azimuthal angles which determine the directions of emission of the decay products of c and d respectively. (In the case of two-particle and three-particle decays these angles are chosen differently [3].) It is easy to show that condition (5) leads to

$$W(\theta_1, \varphi_1, \theta_2, \varphi_2) = W_c(\theta_1, \varphi_1)W_d(\theta_2, \varphi_2), \tag{6}$$

i.e., the angular distributions of the decay products of the formed resonances turn out to be independent.

We note that when resonances are produced the selection rules relative to the isotopic spin and the G-parity in the t-channel usually greatly reduce the number of possible states with shich exchange can be effected, compared with the case of elastic scattering. We can therefore hope that conditions (5) and (6) will be realized in reactions involving production and subsequent decay of resonances at the energies attainable at the present time.

Appreciable correlations between the angular distributions of the products of the decay of K^{*0} and $N^{*++}_{3/2,3/2}$ produced in the reaction $K^+ + p \to K^{*0} + N^{*++}_{3/2,3/2}$ with K^+ having a momentum of 1.96 GeV/c, and of ρ^0 and $N^{*++}_{3/2,3/2}$ produced in the reaction $\pi^+ + p \to \rho^0 + N^{*++}_{3/2,3/2}$ with π^+ having a momentum 3.6 GeV/c. In the case of the first reaction, exchange of states with quantum numbers of the ρ, A_2 (R trajectory) and π mesons is possible, while in the second reaction exchange of A_2 and π mesons is possible. It follows from the foregoing that the presence of correlations denotes absence of factorization of the amplitudes at these energies. This is to be expected, since there are no grounds for assuming that at such low energies we can confine ourselves to the contribution of only a single extreme-right pole in the j-plane.

However, if the model of several Regge poles is correct, then the magnitudes of these correlations should decrease with increasing energy. On the other hand, if an appreciable role is played by branching in the j-plane, then the correlation between the angular distributions of the resonance decay products may not tend to zero as $s \to \infty$.

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$K \to 2\pi$ DECAYS IN THE MODEL WITH ICOSUPLET WEAK COUPLING HAMILTONIAN

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The possible existence of the so-called boson "icosuplet" was discussed by Lee, Okubo, and Schecter [1]. This idea was used later by Biswas, Bose, and Mathur [2], who showed that within the framework of SU(3)-symmetry theory, it is possible to introduce CP violation in the $K \to 2\pi$ decay by ascribing icosuplet transformation properties to the weak-interaction Hamiltonian. In this note, however, we consider a special case of K-meson decays, and we shall neglect effects of CP violation in the analysis. We thus ascribe to the weak-interaction Hamiltonian H_w the following transformation properties:

$$H_w \sim \{10\} + \{10^*\}. \tag{1}$$

We can hope that with this choice of the model Hamiltonian the $K^+ \to \pi^+\pi^0$ decay, which proceeds via a $\Delta T = 3/2$ spurion, calls for an additional analysis compared with the $K_1^0 \to 2\pi$ decay.

We wish to show in this note that it is possible in this case to obtain a connection between the probabilities of these decays. This connection is a generalization of the corresponding rule for $\Delta T = 1/2$. The corresponding spurions responsible for the $K \to 2\pi$ decay transform like states with $Y = -1$, $T = 1/2$ from $\{10\}$ and like states with $Y = -1$, $T = 3/2$ from $\{10^*\}$. Starting from the Hamiltonian (1) given above and from the CPT invariance requirement, we can immediately write out the required decay matrix elements [2]:

$$\langle K^+|H_w|\pi^+\pi^0\rangle = -(3/4\sqrt{2})\, a_{27} \exp(i\delta_2), \tag{2}$$

$$\langle K^0|H_w|\pi^0\pi^0\rangle = \langle \bar{K}^0|H_w|\pi^0\pi^0\rangle = [(1/20)a_{27} + (1/5)a_8]\exp(i\delta_0) - (1/2)a_{27} \exp(i\delta_2), \tag{3}$$