CYCLOTRON RESONANCE IN AN INCLINED MAGNETIC FIELD

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A large number of recent experimental papers are devoted to observation of cyclotron resonance in a constant magnetic field H inclined at an angle to the surface of the metal. At small angles of inclination φ, an explanation was offered for the splitting [1] and doubling [2,3] of the resonance frequency and for the change in the character of the resonance [2-4]. The present note is devoted to the theory of resonance at arbitrary φ, when there is no resonance in the principal approximation in terms of the anomaly [5].

In the next higher approximation in the anomaly, a new type of resonance and of periodic oscillations appears also in a parallel field ($\varphi = 0$), owing to the field and current peaks (similar to those considered in [6], see also below) at depths that are multiples of the orbit diameters D. Participating in the production of these peaks, just as in [6], is a narrow group of electrons having a scatter in D on the order of the effective skin depth $\delta \sim (c^2 D \omega_0^{-2})^{1/3}$ (ω_0 is the plasma frequency and c is the speed of light) with an almost constant cyclotron frequency Ω_e. At frequencies that are multiples of Ω_e a resonance does appear and produces a relative increment ΔZ to the impedance Z, of the order of $\Delta Z/Z \sim \kappa^2[1 - \exp(-i\omega^* T_e - \kappa^{1/2})]^{-2}$, where

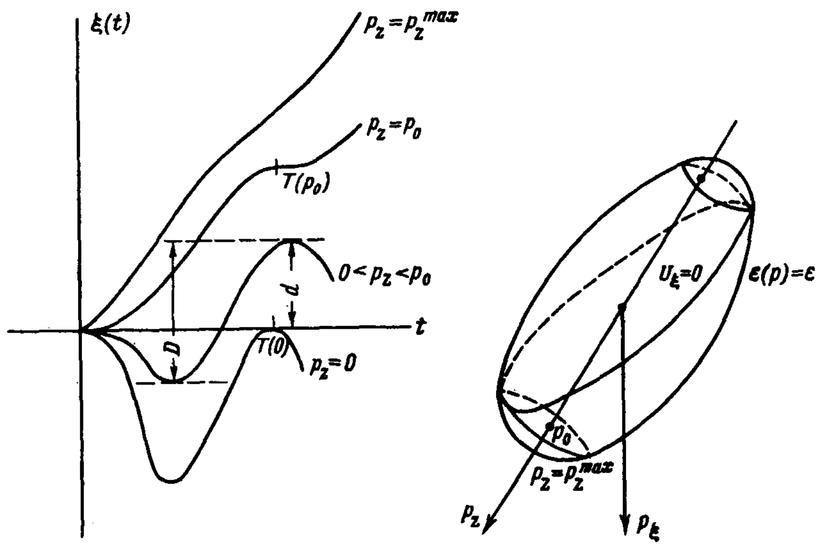
$$\kappa\ (\omega_0 D/c)^{-2/3}, \quad \omega^* = \omega - i/\tau, \quad T = 2\pi/\Omega, \tag{1}$$

τ is the mean-free-path time [5], and ω is the frequency of the alternating field.

Thus, in a parallel field the resonance occurs at frequencies Ω corresponding to the limiting points and the central section (at a definite wave polarization [5]), to the extremal values of the effective mass m, and (owing to the peaks) to the extremal orbit diameters. When the magnetic field is inclined, the first to disappear, even when $\varphi > \delta/D \sim \kappa$, is the resonance on the extremal "noncentral" sections, owing to the electron drift. With increasing inclination of H, the decisive role is assumed by the field peaks (see also [4]), which "pick out" a narrow group of electrons. This alters substantially the shape of the resonance curve [4] (its half-width is of the order of $\gamma + \delta\kappa^{-1/2}$, $\gamma = T/\tau$), whereas the resonance frequency Ω_c remains constant so long as $\varphi \ll \kappa^{1/2}$. When $1 \gg \varphi \gg \kappa^{1/2}$, all that remains is the resonance due to the "new" peaks from the drifting electrons (see below). The amplitude of the resonance drops sharply, its half-width is of the order of γ, and doubling of the resonance frequency (2ω is a multiple of Ω_c), brought about by the Doppler splitting, takes place. A similar change takes place, when φ is varied, in the resonance with the cyclotron frequencies corresponding to D_{max} and D_{min}, except that in the former case the frequency doubling occurs when $\varphi \sim \kappa$.

Let us see how the resonance at the limiting point varies with changing φ. Starting with $\varphi \sim \kappa$, the electrons near the limiting point do not return to the skin layer, and when

$\varphi \gg \kappa$ only electrons that are far from the limiting point can "glide" along the surface with $v_\xi(t) = 0$ (ξ is the normal to the metal surface, t is the time, and $\xi(t)$ is the trajectory of the electron along the ξ axis, see the figure). The last, "effective" electrons (i.e., those accelerated in the skin layer for a long time) near $p_z = p_0$ (p is the quasimomentum; p_0 and t_0 are determined from $v_\xi = 0$ and $dv_\xi/dt = 0$; $z \parallel \underset{\sim}{H}$) are in a special position.



As indicated in [6], the only condition for "focusing" of electrons in the magnetic field at definite depths, and for the resultant field and current peaks, is the selection of orbits in accord with some attribute. Therefore when ξ is a multiple of the "boundary" value $d_0 = d(p_0)$, where d is the path traversed during the period T,

$$d = \int_0^T v_\xi(t)\,dt\,,$$

peaks appear at a depth on the order of the mean free path, and attenuate only because of volume collisions of electrons. If 2ω is a multiple of $\Omega_0 = \Omega(p_0)$, the electrons with $p_z = p_0$ will pass in synchronism through many peaks. (Thus, when $\omega = \Omega_0/2$, the electrons with $p_z = -p_0$, encountering at the depth $\xi = d_0$ the maximum of the field that was carried away by the electrons with $p_z = p_0$ at a time T_0 ago, will fall after a time T_0 again into a field maximum near the surface $\xi = 0$, and the spatial phase shift acquired by the electrons with $p_z = p_0$ is thus automatically eliminated.) The result is a resonance whose damping is due only to the damping of the peaks, and which therefore has a half-width $\sim\gamma = T_0/\tau$. With this,

$$\Delta Z/Z \sim \kappa^{4/3} \sum_{n=1}^{\infty} n^{-2}\exp(-2i\omega^* T_0 n) \sim \kappa^{4/3} A^{-1} \ln A^{-1} \tag{2}$$

$$A = 1 - \exp(-2i\omega^* T_0).$$

(Such a resonance was first pointed out in [2], and for the case $\varphi \ll 1$ see [2,4].)

No resonance is produced only if the interval $2p_0$ degenerates to a point, i.e. (in the case of a plane section $v_\xi = 0$), if p_z is perpendicular to the section $v_\xi = 0$. In the case of isotropic dispersion this corresponds to H perpendicular to the metal surface.

An experimental investigation of this resonance (including orbit cutoff [7]) makes it possible to determine the effective mass and the area S of the Fermi-surface section as a function of p_z for any direction of z (using the fact that $d = (c/eH)(\partial S/\partial p_z)_\epsilon$), i.e., the same information that can, in principle, be obtained from quantum cyclotron resonance [8]. When $\varphi \ll 1$ it is possible to determine in this manner the Gaussian curvature [5] K and (from the Doppler splitting) the velocity v on the Fermi surface. For the same reasons as for $\varphi = 0$ (see [5]), polarization of the electric field along the velocity at the limiting point is necessary when $\varphi \ll 1$.

It is most useful to compare the same Fermi-surface characteristics given by different experiments. The accuracy of the agreement between them may be evidence of the degree of accuracy obtained by introducing quasiparticles - conduction electrons - in the metal.

Resonance for any φ at $2\omega = n\Omega$ (n an integer) will be assured by sections in which d has an extremum (if they exist). Resonance appears on such sections when $\varphi \gg \kappa$; when $\varphi \sim 1$, $\Delta Z/Z \sim \ln A^{-1}$, where T in formula (2) is taken on these sections.

Open sections give $\Delta Z/Z \sim \kappa A^{-1}$ with Ω corresponding to diameters $D = \xi_{max} - \xi_{min}$ which are extremal in p_z (the minimum and maximum of ξ are taken with respect to t within the limits of a single period, see the figure). We note that resonance in an inclined field can be sharper than when $\varphi = 0$, since τ is not reduced by collisions with the irregularities on the surface. Naturally, none of the foregoing results pertain to singular angles φ (at which, for example, the extremum of d vanishes, or where $d' = 0$ and $d'' = 0$ simultaneously, or else where resonance frequencies from different sections coincide).

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