Substituting the obtained values of k_i in (4a), we can easily see that the remaining φ_{ij} are small compared with $\varphi_{k-1,k}$; this justifies the assumption made above.

We present without proof some consequences of (9):

1. When $\gamma < -1$, the bound states of L particles have an energy lower than that of L - 1 bound particles plus one free particle (of course, for the same value of u).

2. When $\gamma > 1$, the bound states of L particles have an energy higher than that of L - 1 bound particles plus one free particle.

3. When $|\gamma| < 1$, the bound states of L particles fall in the continuous spectrum (with exception of L = 2).

4. Summation over the permutations yields for the wave function of the bound state (3) only one term, $\exp i(k_1n_1 + \ldots + k_Ln_L)$.

5. When $|\gamma| > 1$ the complex is rigid, i.e., the probability of a hole appearing in the middle of the complex is negligible. Holes have a noticeable probability of occurrence only at the ends of the complex.

6. When $|\gamma| \leq 1$ the complex becomes brittle, i.e., states with a hole inside the complex are possible.

7. As $L \to \infty$, we get $E = L\gamma$ for all γ and the particle density in the complex tends to unity.

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ANOMALOUS DIFFUSE SMALL-ANGLE MOSSBAUER SCATTERING IN CRYSTALS

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Bragg scattering of x rays in crystals is known to be accompanied by anomalous diffuse scattering in a narrow angle interval near the diffraction maximum (which corresponds to a wave-vector direction $\mathbf{k}_1 = \mathbf{k} + \mathbf{K}$, where $\mathbf{K}$ is the reciprocal-lattice vector). The differential scattering cross section is proportional here to $1/\kappa^2$, where $\kappa = \mathbf{k}' - \mathbf{k}_1$ (cf., e.g., [1]). Physically this is due to the periodicity of the phonon spectrum in the reciprocal-lattice space, whereby particle scattering in a regular crystal can always excite phonons of extremely low frequencies and yet effect an appreciable momentum transfer.

One the other hand, although low-frequency phonons are excited in small-angle scattering, there is no anomalous scattering, since the scattering cross section is proportional to the square of the momentum transfer. This result seems perfectly general at first glance.

It turns out, however, that a highly unique situation can arise in the case of resonant scattering.

For potential scattering, the effective interaction time is small compared with $1/\omega_D$, where ω_D is the characteristic frequency of the phonon spectrum. By virtue of this, the atoms receive a recoil momentum equal to $\underset{\sim}{k} - \underset{\sim}{k}'$. On the other hand, in resonant scattering, if the width Γ of the resonance level is small compared with ω_D, the atoms acquire a recoil momentum $\underset{\sim}{k}$ following absorption and a recoil momentum $-\underset{\sim}{k}'$ following emission. Therefore small-angle scattering is not cut off at a total momentum transfer $\underset{\sim}{k} - \underset{\sim}{k}'$. This is precisely the situation in resonant scattering of γ quanta by Mossbauer nuclei, for which $\Gamma/\omega_D \ll 1$ always. The amplitude of the coherent resonant scattering of γ quanta by a crystal of limited dimensions with emission (absorption) of a phonon with wave vector q and with branch number α is [2,3]:

$$f_{\lambda\lambda'}(k;\ k',q,\alpha) = -\frac{\Gamma_1'}{2k}\phi_{\lambda\lambda'}(\theta,\theta')\sum_{s}\sum_{\{n''\}} e^{i(k-k')R_s}$$

$$\times \frac{(e^{iku_s})_{\{n\}\{n''\}}(e^{-ik'u_s})_{\{n''\}\{n'\}}}{E_k - E_o - \sum_{\beta}\omega_{\beta}(n_{\beta}'' - n_{\beta}) + i\Gamma/2}. \tag{1}$$

Here $\varphi_{\lambda\lambda'}(\theta, \theta')$ characterizes the dependence of the scattering amplitude on the polarizations and directions of the incident (λ, θ) and scattered (λ', θ') γ quanta and $\Gamma_1' = \Gamma_1(2I + 1)/2(2I_0 + 1)$. All other symbols are standard.

The state $\{n'\}$ differs from $\{n\}$ only in that $n_{q\alpha}' = n_{q\alpha} \pm 1$. In the sum over $\{n''\}$ the main terms are those with $\{n''\} = \{n\}$ and $\{n''\} = \{n'\}$. It is easy to verify that the contribution of the remaining terms in this sum will be small, of the order of Γ/ω_D. Taking this into consideration, we obtain after simple transformations

$$f_{\lambda\lambda'}(k; k', q, \alpha) = i\frac{\Gamma_1'}{2\kappa}\phi_{\lambda\lambda'}(\theta,\theta')\, e^{-1/2[Z(\underset{\sim}{k}) + Z(k')]}\cdot$$

$$\times \frac{1}{\sqrt{2MN\omega_{qa}}}\sqrt{\bar{n}_{q\alpha} + 1/2 \pm 1/2}\left\{\frac{k'e(q,\alpha)}{E_k - E_o + i\Gamma/2} - \frac{k\,e(q,\alpha)}{E_k - E_0 \mp \omega_{q,\alpha} + i\Gamma/2}\right\} \tag{2}$$

$$\times \sum_s e^{i(k - k' \mp q)R_s}.$$

Here $\exp[-Z(k)]$ is the probability of the Mossbauer effect; ω_{qa}, $e(q, \alpha)$, and n_{qa} are the frequency, polarization, and **average** occupation number of the phonon q, α. For **simplicity**, we have considered a **monatomic** crystal, and here N is the number of atoms in the crystal and M the mass of the atom.

The sum over s in (2) leads to $\underset{\sim}{k}' = \underset{\sim}{k} \mp \underset{\sim}{q}$. For the small-angle scatterign region of interest to us we can neglect the difference between k' and k in the factor $k'e(q, \alpha)$ and in

$Z(k')$. In addition, in this limit we have $\varphi_{\lambda\lambda'}(\theta, \theta') = \delta_{\lambda\lambda'}$, and for finite temperatures $\check{n}_{qa} \approx \tilde{n}_{qa} + 1 \approx T/\omega_{qa}$.

We go over from the scattering amplitude to the differential cross section per nucleus:

$$\frac{d\sigma}{d\Omega_{k'}} = \frac{1}{4\pi}\ \sigma_t(E_k)\frac{\Gamma_1'}{\Gamma}\ e^{-2Z(k)}\frac{T}{2M}\sum_{\pm}\sum_{a}\frac{|k\,e(k'-k,a)|^2}{(E_k - E_0 \mp \omega_{qa})^2 + \Gamma^2/4}, \tag{3}$$

where $\sigma_t(E_k)$ is the total cross section for scattering by an individual nucleus,

$$\sigma_t(E_k) = \frac{\pi}{k^2}\frac{\Gamma_1'\Gamma}{(E_k - E_0)^2 + \Gamma^2/4}; \qquad E_{k'} = E_k \mp \omega_{qa}.$$

By virtue of the fact that the phonon energy is negligibly small compared with the γ-quantum energy, the transferred momentum $q = k' - k$ is perpendicular to k. Therefore $d\Omega_{k'} = d^2q/k^2 = q\,dq\,d\varphi/k^2$, and we can directly go over from (3) to the total cross section, bearing in mind that the main contribution is connected with small values of q:

$$\sigma = \sigma_t(E_k)\frac{\Gamma'}{\Gamma}\ e^{-2Z(k)}\frac{T}{4M}\sum_{\alpha}\int\frac{d\phi}{2\pi}\frac{|ne(\phi,\alpha)|^2}{C_\alpha^2(\phi)}\Big\{\ln\frac{[C_\alpha(\phi)q_0]^2}{\Delta^2 + \Gamma^2/4} + \frac{4\Delta}{\Gamma}\tan^{-1}\frac{2\Delta}{\Gamma}\Big\}. \tag{4}$$

Here $\Delta = E_k - E_0$, q_0 is a wave vector on the order of the limiting phonon vector, and $n = k/k$. The $C_\alpha(\varphi)$ are determined from the relation $\omega_{q\alpha} = C_\alpha(\varphi)q$. Expressions (3) and (4) solve our problem completely. (If the resonant-pair concentration η differs from unity, both the differential and the integral scattering cross sections are multiplied by η.) It follows from (3) that in the resonant case anomalous diffuse small-angle scattering does indeed take place. For small angles actually attainable in investigations of the differential scattering cross section $\omega_{q\alpha} \gg |\Delta|$, Γ and $d\sigma/d\Omega_{k'} \sim 1/q^2$.

Such a sharp dependence on the momentum transfer makes it easy to separate the anomalous diffuse small-angle scattering. It is interesting that at extremely low q the differential cross section tends to a finite limit (this is very important for the integral cross section). Physically this is connected with the fact that when $q < \Gamma/C_s$ the corresponding phonon frequencies become smaller than Γ, and for such oscillations the scattering is nearly potential.

It is interesting that the use of the Mossbauer effect makes possible direct separation of the radiation scattered through small angles (down to $q \sim \Gamma/C_s$), from the purely resonant radiation that passes through without being scattered. To this end, obviously, it is sufficient to measure the attenuation of the integral intensity in a resonant absorber as a function of the velocity of the latter (see [4]). Thus, it becomes possible to analyze anomalous small-angle scattering also when the integral scattering cross section (4) is measured.

We note in conclusion that in scattering by very small single crystals, when the transverse dimension is $L < 2\pi C_s/\Gamma$, the scattering cross section turns out to be bounded, inasmuch as the wave vector cannot be smaller than $q_{min} = 2\pi/L$. The second term in the square brackets in (4) then vanishes, and the first term is replaced by $\ln(q_0/q_{min})^2$.

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FEASIBILITY OF INVESTIGATING A PINCH DISCHARGE BY USING ITS INTRINSIC STIMULATED EMISSION

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The existence of negative-temperature states in a high-temperature plasma of a strong-current pinch discharge has been demonstrated by us in an earlier paper [1], where stimulated emission was generated for the first time in a plasma of such a discharge. We report here the use of this phenomenon to investigate the cumulation of a pinch discharge. To this end we measured the time correlation between the stimulated-emission pulse and the current pulse at the instant of discharge cumulation.

The discharge was produced in a quartz tube of 40 mm inside diameter and 1 m length. The electrodes were tantalum rings of 40 mm diameter. The energy sources were 0.01, 0.1, and 0.4 μF capacitors charged to as much as 45 kV. The maximum discharge current was 20 kA at a discharge duration 2 μsec. The current density at the instant of cumulation reached 50 - 75 kA/cm^2. To observe the stimulated emission pulse, confocal dielectric-coated mirrors designed for the emission-wavelength (λ = 4500 - 5000 Å) were mounted at the ends of the discharge chamber. The working gas was spectrally pure argon. To facilitate probe measurements, the discharge-chamber diameter was increased (to 40 mm). The optimal pressures for generation were then appreciably lower than in the cited investigation [1], amounting to 2×10^{-3} - 5×10^{-4} mm Hg (see Fig. 1).

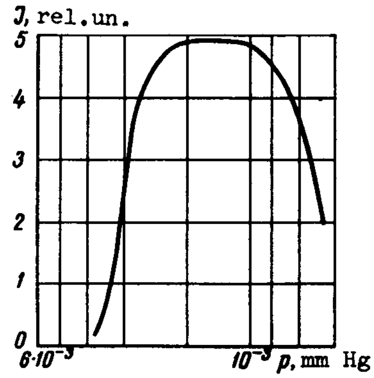

Fig. 1. Stimulated-emission intensity vs. initial pressure.

To establish the time correlation between the generation pulse and the current at the instant of cumulation, measurements were made with a Rogowski loop placed in a glass tube bent in the form of a ring, with inside and outside diameters 7 and 12 mm. It follows from such measurements that at the instant of cumulation the current is concentrated in the axial part of the discharge chamber, and there is none in the region adjacent to the wall. Gener-