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CRITICAL TEMPERATURE OF SMALL SUPERCONDUCTORS

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It is known [1] that the critical temperature of sufficiently small superconductors (non-annealed films) increases with decreasing dimensions. Attempts were made to explain this effect theoretically. Kirzhnits and Maksimov [2] related the growth of the transition temperature with an increase of the effective constant for the interaction between the electrons in the surface layer, due to the Rayleigh waves. Kresin and Tavger [3] attempted to attribute this effect to quantization of the transverse momentum of the electrons in thin films, a quantization which is in fact nonexistent by virtue of the diffuse reflections of the electrons from the surfaces of even very "good" films.

We shall show below that the growth of the critical temperature of small samples can be explained within the framework of the BCS superconductivity theory without making use of any new electron-pairing mechanism whatsoever.

The author has shown in [4] that the critical temperature (and to an equal extent also the temperature dependence of the ordering parameter Δ) does not depend on the shape or dimensions of the sample. The limitation mentioned in that paper (sample dimensions larger than v_F/ω_D), connected with the fact that momentum cutoff in Bardeen's four-fermion Hamiltonian leads to a spatial smearing of the interaction at distances $\sim v_F/\omega_D$ (see, e.g., [5]), is in fact insiginificant. Indeed, if we start from the more realistic Frohlich Hamiltonian and use the method developed by Eliashberg [6], we can show that the cutoff in the equation

$$\Delta(r) = |\lambda| \sum_{\omega} F_\omega(r, r) \tag{1}$$

should be carried out during the calculation not with respect to the momenta (or energies), but with respect to the frequencies at $\omega \sim \omega_D^*$. This means that the interaction of the electrons via the phonon field, which is responsible for the superconductivity, is local and not instantaneous, this being connected with the low propagation velocity of the lattice excitations. Indeed, the lifetime of the virtual excitation $\tau \sim 1/\omega_D$ is relatively large, but during that time the excitation can propagate only over a distance on the order of the lattice constant $a \sim \tau c$.

The transition temperature can be determined by finding the maximum temperature at which a nontrivial solution exists for the integral equation

$$\Delta(r) = |\lambda| T \sum_{|\omega| < \omega_D} \int G_\omega(r, r') G_{-\omega}(r, r') \Delta(r') \, dr'. \tag{2}$$

We employ, as usual, a temperature technique [5], so that $\omega = (2n + 1)\pi T$. Here G_ω is the temperature Green's function of the normal electron, and λ is the four-fermion interaction constant. Expanding, as in [3], G_ω in terms of real eigenfunctions of the electron in the sample under consideration, $\psi_n(r)$, we get

$$\Delta(r) = |\lambda| T \sum_{|\omega| < \omega_D} \sum_{n, m} \frac{\psi_n(r)\psi_m(r)}{(\xi_n - i\omega)(\xi_m + i\omega)} \int \psi_n(r')\psi_m(r')\Delta(r')dr', \qquad (3)$$

where ξ_n is the energy of the corresponding state, reckoned from the Fermi level, and the summation is over all the states of the electron.

Substituting in the right side of (3) the constant value of the parameter $\Delta(r) = \Delta$, we obtain

$$\Delta(r) = |\lambda| \nu \ln \frac{2\gamma\omega_D}{\pi T_c} \cdot \Delta \overline{\psi_n^2(r)}, \qquad (4)$$

where ν is the density of states at the Fermi level, and the square of the wave function is averaged over all states near the Fermi surface, lying in an energy interval that is narrow compared with T_c. If we neglect boundary effects, this average is equal to $1/V$ and we obtain the result of [4], viz., that the transition temperature is independent of the shape and dimensions of the sample.

If we take boundary effects into account, then ψ_n^2 decreases near the boundary over distances of the order of $1/p_F$, and vanishes on the surface of the sample. Since the integral of ψ_n^2 over the volume remains equal to unity, this increases the mean square inside the sample:

$$\overline{\psi_n^2(r)} = (V - \frac{\alpha S}{p_F})^{-1}, \qquad (5)$$

where V and S are the volume and surface of the sample, and the coefficient $\alpha \sim 1$ and depends on the boundary conditions. In particular, for a plane surface $\alpha = \pi/4$, and

$$\overline{\psi_n^2(r)} = \begin{cases} (V - \dfrac{\pi S}{4p_F})^{-1} & \text{inside sample} \\[2ex] (V - \dfrac{\pi S}{4p_F})^{-1}(1 - \dfrac{\sin 2p_F z}{2p_F z}) & \text{near surface} \end{cases} \qquad (5a)$$

(the z axis is perpendicular to the surface). The latter result is valid if the plane sections of the sample surface do not exceed $1/p_F$.

We must furthermore recognize that the presence of the boundary shifts the Fermi level $E_F = p_m^2/2m$. The effective volume, i.e., the region accessible to the electrons, is smaller than the volume of the sample, in accord with (5), by an amount $\alpha S/p$, from which we get an equation for the shift of the Fermi level

$$\int_o^{P_F} (1 - \frac{aS}{pV}) p^2 dp = \int_o^{P_o} p^2 dp, \tag{6}$$

where p_0 is the Fermi momentum in an infinite volume. Hence

$$\nu = \nu_o (1 - aS / 2p_o V), \tag{7}$$

where ν_0 is the state density for a sample of infinite size.

The final equation for the transition temperature can thus be written in the BCS form:

$$1 = |\lambda| \nu_{ef} \ln \frac{2\gamma\omega_D}{\pi T_c}, \tag{8}$$

where

$$\nu_{ef} = \nu \overline{\psi_n^2} V = \nu_o (1 + \frac{a}{2} \frac{S}{P_o V}) \tag{9}$$

and thus takes account of both effects noted above.

Physically ν_{ef} has the meaning of the state density near the Fermi surface, divided by the effective volume.

From (8) and (9) we get the correction to the transition temperature:

$$\frac{\Delta T}{T_c} = \frac{a}{2} \frac{S}{P_o V} \ln \frac{2\gamma\omega_D}{\pi T_c}. \tag{10}$$

It is easy to carry out similar calculations for arbitrary temperatures. The temperature dependence of the ordering parameter $\Delta(T)$ remains the same as before (in particular, $\Delta(0) = \pi T_c/\gamma$), except that it is necessary to use in all the corresponding formulas the critical temperature defined by (8) and (9).

For ordinary-metal samples with thickness on the order of 10^{-6} cm, formula (10) predicts an increase of 10 - 15% in the critical temperature, in good agreement with experimental data on non-annealed thin films. Such films consist apparently of a large number of small crystallites with dimensions on the order of or smaller than the film thickness. When the films are annealed, these crystallites become "sintered" and this leads to an appreciable decrease in the total surface and consequently to a corresponding decrease in the transition temperature, as is indeed observed in the experiment.

In conclusion, the author thanks A. A. Abrikosov for an interesting discussion and for useful remarks.

Note. After this letter was written, Abeles et al. [7] published a very interesting paper in which the dimensions of the crystallites making up the investigated films were determined for the first time. For indium and tin, with crystallite dimensions 110 Å, the transition temperature was found to rise by 10%, which is in good agreement with (10) ($\ln[2\gamma\omega_D/\pi T_c$ equals 0.36^{-1} and 0.31^{-1} respectively for these metals). For aluminum, the

variation of T_c with L agrees with (8) - (9), but the observed values of $\Delta T/T_c$ are apparently four times larger. It is possible that this is due to the strong anisotropy and the continuous structure of the Fermi surface of aluminum: formula (7), which is derived for the isotropic model, is not applicable here, for the state density apparently increases with decreasing sample dimensions, leading to an even greater growth of the effective density than predicted by (9).

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SELF-FOCUSING OF LIGHT IN THE KERR EFFECT

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Light can become self-focused in a liquid via the Kerr effect [1]. The self-focusing threshold of circularly polarized light (in the light channel) is four times the threshold of linearly polarized light [2]. We shall discuss here the question of self-focusing of elliptically polarized light (and of circular polarization as a particular case). Under certain conditions, the light channels produced for such light will have not elliptical but linear polarization. This, in particular, lowers the self-focusing threshold for circular polarization to a value one-half that obtained in [2]. This question has a direct bearing on the problem of laser-beam stratification [3] in the case of elliptic polarization of light in liquids, as well as in solids, where electrostriction is the stronger self-focusing mechanism.

The dielectric tensor in the electric field is [2,4]:

$$\epsilon_{ij} = \epsilon_0 + 3aE_iE_j - a\delta_{ij} \cdot \sum_k E_k^2 \ . \tag{1}$$

Assume that elliptically polarized light

$$E_y = E_1 \cos(\omega t - kx); \ E_z = E_2 \cos(\omega t - kx \pm \frac{\pi}{2})$$

propagates along the x axis. Then contributions to ϵ_{ij} are made only by the time-averaged