variation of T_c with L agrees with (8) - (9), but the observed values of $\Delta T/T_c$ are apparently four times larger. It is possible that this is due to the strong anisotropy and the continuous structure of the Fermi surface of aluminum: formula (7), which is derived for the isotropic model, is not applicable here, for the state density apparently increases with decreasing sample dimensions, leading to an even greater growth of the effective density than predicted by (9).

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SELF-FOCUSING OF LIGHT IN THE KERR EFFECT

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Light can become self-focused in a liquid via the Kerr effect [1]. The self-focusing threshold of circularly polarized light (in the light channel) is four times the threshold of linearly polarized light [2]. We shall discuss here the question of self-focusing of elliptically polarized light (and of circular polarization as a particular case). Under certain conditions, the light channels produced for such light will have not elliptical but linear polarization. This, in particular, lowers the self-focusing threshold for circular polarization to a value one-half that obtained in [2]. This question has a direct bearing on the problem of laser-beam stratification [3] in the case of elliptic polarization of light in liquids, as well as in solids, where electrostriction is the stronger self-focusing mechanism.

The dielectric tensor in the electric field is [2,4]:

$$\epsilon_{ij} = \epsilon_0 + 3aE_iE_j - a\delta_{ij}\cdot\sum_k E_k^2 . \tag{1}$$

Assume that elliptically polarized light

$$E_y = E_1\cos(\omega t - kx);\ E_z = E_2\cos(\omega t - kx \pm \frac{\pi}{2})$$

propagates along the x axis. Then contributions to ϵ_{ij} are made only by the time-averaged terms $E_i E_j$. We get

$$\epsilon_{xx} = \epsilon_o - \frac{a}{2}(E_1^2 + E_2^2), \quad \epsilon_{yy} = \epsilon_o + aE_1^2 - \frac{a}{2}E_2^2,$$

$$\epsilon_{zz} = \epsilon_o - \frac{a}{2}E_1^2 + aE_2^2. \tag{2}$$

All the remaining ϵ_{ij} components are equal to zero.

We shall show that if the condition

$$E_2 > \sqrt{2}\,E_1 \tag{3}$$

is satisfied, only a channel in which the light is linearly polarized can be produced. To this end, let us consider the dynamics of cutting through the channel. Let the beam intensity in some region exceed the average value. Then ϵ_{zz} in this region is larger than in the remaining space, and ϵ_{yy} is smaller. This leads to capture of rays polarized along the z axis (z-rays), and to departure of some of the rays polarized along the y axis (y-rays) from the region under consideration. But both the increase in E_2 and the decrease in E_1 lead to further growth of ϵ_{zz} and decrease of ϵ_{yy} (see (2)). Thus, if condition (3) is satisfied, a channel will be produced for the z-rays, and the y-rays will leave this channel. In turn, if the intensity of the y-rays is sufficient to produce a channel, they will become self focused in a geometrically-independent channel with y-polarization. (The formation of a two-layer cylindrical region, in which the self-focused z-rays are surrounded by an outer layer of self-focused y-rays, appears unlikely in this case even if the light intensity is very high. A certain amount of fluctuation will apparently lead to formation of a geometrically-independent channel for the y-rays.) We note that actually condition (3) for the incident light is too stringent. Even if the y- and z-rays can initially become focused together, differences in the self-focusing channel diameters, due to differences between ϵ_{yy} and ϵ_{zz}, will make it possible to satisfy condition (3) inside the channel for the z-rays, starting at some instant of time, and the y-rays will then leave the z-ray channel. A certain amount of fluctuation leads to formation of an independent channel for the y-rays, too, if their intensity is sufficiently high. (In general, occurrence of a two-layer cylindrical region containing z-rays on the inside and y-rays on the outside is perfectly probable if the y- and z-rays are initially focused together, provided the light-beam power is high.)

Let us consider now the case of circularly polarized incident light. Let the power of the light flux be lower than the threshold for formation of a channel with circular polarization, but let the amplitude $E_1 = E_2$ be sufficient to permit self-focusing of linearly polarized light of amplitude E_1. We shall show that two geometrically-independent channels with linear polarization are produced in this case, i.e., the self-focusing threshold decreases by one-half compared with that determined in [2]. Indeed, although the y- and z-rays cannot be self-focused together, the presence of uniformly distributed z-rays does not interfere in any way with the start of self-focusing of the y-rays. To the contrary, a decrease in the intensity of the z-rays in the self-focusing region of the y-rays (because ϵ_{yz} decreases, in accord with (2), with increasing E_1) will, again in accord with (2), enhance the latter by increasing ϵ_{yy} (owing to the decrease of E_2). Let us point out immediately that variations in the density of the incident light flux over the cross section of the beam do not affect the light channels in this case, since the threshold for formation of a channel with circular polarization has not yet been reached. A self-focusing center can arise only when the equality $E_1 = E_2$ is violated in some part of the laser beam. If this equality holds, then thermal fluctuations of ϵ_{ij} can serve as a self-focusing center. Assume that ϵ_{yy} fluctuates in some region in such a way that $d\epsilon_{yy} > 0$ (but $d\epsilon_{zz} \leq 0$). Then the number of y-rays in this region increases, and this reinforces the fluctuations $d\epsilon_{yy}$ since the y-ray power exceeds the threshold for the self-focusing of linearly polarized light. Accordingly, the z-ray intensity decreases in this region after a certain time, and this contributes, as shown above, to the self-focusing of the y-rays. This produces, under the assumptions made, a channel for the self focusing of y-rays, with the z-rays going out of this channel. The channel for self-focusing of the z-rays is produced elsewhere.

If the power of the light beam exceeds the threshold for the formation of a channel with circular polarization, then such channels can be produced in principle. But channels with linear polarization can also be produced, say as a result of violation of the equality $E_1 = E_2$ in some region of the laser beam. In addition, even if production of a channel with circular polarization has started, it is unavoidably accompanied by strong fluctuations of ϵ_{ij}. These fluctuations can defocus the z-rays and simultaneously focus the y-rays in some region. Repeating the reasoning presented above, we can readily see that channels with linear polarization can arise as the self-focusing process evolves. (The two-layer cylindrical channel described above can also occur in principle.)

Thus, self-focusing of elliptically polarized light results in many cases in formation of channels with linear polarization. This effect can be investigated experimentally by tracing with the aid of polaroids the intensity distribution of differently-polarized light over the cross section of the beam.

It is interesting to note that inasmuch as the decisive role in the cutting-through of the light channel is always played by the Kerr effect [2], the altter can be the cause of stratification of a laser beam even in solids, where the electrostriction effect in a stationary channel is much stronger than the Kerr effect. However, in the stationary mode the polarization inside the channels will already become elliptical and not linear, for beams of both polarizations will be equally well maintained in each of the channels after the electrostriction begins to play the decisive role.

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SUM RULE FOR π^0, η, AND X^0 MESON INTERACTION CONSTANTS

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A principle postulating minimal signularity of interaction was introduced in [1] within the framework of an axiomatic approach. A consisten application of this principle to scattering amplitudes makes it possible to establish the required number of subtractions in the dispersion integrals for invariant amplitudes.

It can be shown, in particular, that out of the six invariant amplitudes $T_i(s, t)$ ($i = 1, 2, \ldots, 6$), in terms of which the total γN-scattering amplitude is expanded [2,3], T_2, T_4, and T_6 satisfy subtractionless dispersion relations in the s-channel, while the amplitude T_5 approaches a constant limit asymptotically (as $s \to \infty$).

The dispersion relations for T_6 at the point $s = m^2$, $t = 0$ yield the well known sum rule for the squares of the anomalous magnetic moments of the nucleon [5,6]. The possible existence of this rule was first pointed out by Lapidus and Chou Kuang-chao [7].

In this note we derive a sum rule for the amplitude T_5 and attempt to relate it to the constants of the π^0, η, and X^0 mesons. Since $T_5(s, t)$ asymptotically approaches a constant, and in view of the low-energy limit (The Low theorem [7]), we have

$$T_5(m^2, o) = \frac{e^2}{2m}\left(2\mu_p' \frac{1 + \tau_3}{2} + \lambda^2\right) = \Lambda_5 + \frac{2}{\pi}\int_{m^2}^{\infty} \frac{\operatorname{Im} T_5(x, o)}{x - m^2}\, dx, \qquad (1)$$

where $\lambda = \mu_p'[(1 + \tau_3)/2] + \mu_n[(1 - \tau_3)/2]$; μ_p' and μ_n are the anomalous magnetic moments of the proton and neutron; m is the nucleon mass; $e^2/4\pi = 1/137$; T_5 is the limiting value of the amplitude at infinity (in the s-channel).

We note that $T_5(s, 0) = \frac{1}{2}[T_1(s, 0) + T_3(s, 0)]$ when $t = 0$. It follows therefore that the dispersion relations for $T_1 + T_3$ have a form similar to (1) for forward scattering.

The constant Λ_5 can be expressed in terms of a dispersion integral of the contribution $A_{III}^{(5)}(t)$, which is independent of s, to the absorptive part of the amplitude T_5 in the