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SUM RULE FOR π^0, η, AND X^0 MESON INTERACTION CONSTANTS

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Submitted 10 October 1966
ZhETF Pis'ma 5, No. 2, 64-67, 15 January 1967

A principle postulating minimal signularity of interaction was introduced in [1] within the framework of an axiomatic approach. A consisten application of this principle to scattering amplitudes makes it possible to establish the required number of subtractions in the dispersion integrals for invariant amplitudes.

It can be shown, in particular, that out of the six invariant amplitudes $T_i(s, t)$ ($i = 1, 2, \ldots, 6$), in terms of which the total γN-scattering amplitude is expanded [2,3], T_2, T_4, and T_6 satisfy subtractionless dispersion relations in the s-channel, while the amplitude T_5 approaches a constant limit asymptotically (as $s \to \infty$).

The dispersion relations for T_6 at the point $s = m^2$, $t = 0$ yield the well known sum rule for the squares of the anomalous magnetic moments of the nucleon [5,6]. The possible existence of this rule was first pointed out by Lapidus and Chou Kuang-chao [7].

In this note we derive a sum rule for the amplitude T_5 and attempt to relate it to the constants of the π^0, η, and X^0 mesons. Since $T_5(s, t)$ asymptotically approaches a constant, and in view of the low-energy limit (The Low theorem [7]), we have

$$T_5(m^2, o) = \frac{e^2}{2m}\left(2\mu_p' \frac{1 + \tau_3}{2} + \lambda^2\right) = \Lambda_5 + \frac{2}{\pi}\int_{m^2}^{\infty} \frac{\mathrm{Im}\, T_5(x, o)}{x - m^2}\, dx, \tag{1}$$

where $\lambda = \mu_p'[(1 + \tau_3)/2] + \mu_n[(1 - \tau_3)/2]$; μ_p' and μ_n are the anomalous magnetic moments of the proton and neutron; m is the nucleon mass; $e^2/4\pi = 1/137$; T_5 is the limiting value of the amplitude at infinity (in the s-channel).

We note that $T_5(s, 0) = \frac{1}{2}[T_1(s, 0) + T_3(s, 0)]$ when $t = 0$. It follows therefore that the dispersion relations for $T_1 + T_3$ have a form similar to (1) for forward scattering.

The constant Λ_5 can be expressed in terms of a dispersion integral of the contribution $A_{III}^{(5)}(t)$, which is independent of s, to the absorptive part of the amplitude T_5 in the t-channel

$$\Lambda_5 = \frac{1}{\pi} \int_0^\infty \frac{A_{III}^{(5)}(t')}{t'} \, dt' . \tag{2}$$

This result can be proved rigorously by merely making the additional assumption that the amplitude $T_5(s, t)$ satisfies a single dispersion relation in the t-channel and tends to zero when $|s|$ and $|t| \to \infty$ within the limits of the physical region.

If it is assumed that the π^0, η, and X^0 mesons are "elementary" particles then

$$\Lambda_5 \cong \frac{a_{\pi^\circ}}{\mu^2_{\pi^\circ}} + \frac{a_\eta}{\mu^2_\eta} + \frac{a_{X^\circ}}{\mu^2_{X^\circ}}, \tag{3}$$

where α_{π^0}, α , and α_{X^0} are the residues at the π^0-, η-, and X^0-meson poles, and are equal to

$$a_{\pi^\circ} = \pm\, 8\pi\mu_{\pi^\circ}[\Gamma_{\pi^\circ\to 2\gamma}/\mu_{\pi^\circ} \quad g^2_{\pi NN}/4\pi]^{1/2},$$

$$a_\eta = \pm 8\pi\mu_\eta[\Gamma_{\eta\to 2\gamma}/\mu_\eta \cdot g^2_{\eta NN}/4\pi]^{1/2},$$

$$a_{X^\circ} = \pm 8\pi\mu_{X^\circ}[\Gamma_{X^\circ\to 2\gamma}/\mu_{X^\circ} \quad g^2_{X^\circ NN}/4\pi]^{1/2}.$$

For comparison with experiment, let us consider Compton scattering by a proton and let us take into account the contribution made to Im $T_5(x, 0)$ by the photoproduction of single pions only. We shall retain in the photoproduction amplitude the S-wave, the partial waves corresponding to the first, second, and third resonances, and also the delay term. Evaluation of the integral in (1) leads to the expression

$$\frac{0{,}187}{m} \cong \frac{a_{\pi^\circ}}{\mu^2_{\pi^\circ}} + \frac{a_n}{\mu^2_\eta} + \frac{a_{X^\circ}}{\mu^2_{X^\circ}} . \tag{4}$$

Let us discuss several possibilities.

I. We assume the π^0-meson lifetime to be $\tau_{\pi^0} = 10^{-16}$ sec. Then, from symmetry theory, the lifetime of the η meson is of the order of 10^{-18} sec. In addition, symmetry theory gives opposite signs [9] for the matrix elements of the decays $\pi^0 \to 2\gamma$ and $\eta \to 2\gamma$, and consequently also for the residues α_{π^0} and α_η. We shall assume the interaction constant $g_{\eta NN}$ to be equal to $g_{\pi NN}$. Since the η and X^0 mesons differ from each other only in mass, it is natural to assume that α_η and α_{X^0} have the same sign. It is then easy to show that α_{π^0} in (4) should be negative (this agrees with the results of [10]). Taking all the foregoing into account, we get

$$\frac{g^2_{X^\circ NN}}{4\pi}\, \Gamma_{X^\circ \to 2\gamma} \cong 7{,}5\cdot 10^{-2}\ \text{MeV} \tag{5}$$

II. We now assume that the X^0 meson is a reggeon. Then only the π^0 and η mesons will contribute to (4). Putting $\tau_{\pi^0} = 10^{-16}$ sec we find that $\alpha_\eta > 0$ irrespective of the sign of the residue α_{π^0}. If $\alpha_{\pi^0} < 0$, then $\tau_\eta = 1.24 \times 10^{-19}$ sec (it is assumed here, as before, that $g_{\eta NN} = g_{\pi NN}$).

III. If it is assumed that only the π^0 meson is an "elementary" particle, then (4) takes the form

$$\frac{\alpha_{\pi^0}}{\mu^2_{\pi^0}} \cong \frac{0,187}{m}. \tag{6}$$

That is to say, we find that $\alpha_{\pi^0} > 0$. Expression (6) corresponds to $\tau_{\pi^0} = 0.63 \times 10^{-16}$ sec, whereas the latest experimental data [11] yield $\tau^{exp}_{\pi^0} = (0.73 \pm 0.105) \times 10^{-16}$ sec.

Since, first, the results of [10] favor the choice $\alpha_{\pi^0} < 0$, and second, the lifetime τ_η in variant II is smaller by one order of magnitude than that expected from symmetry theory, it must apparently be assumed that π^0, η, and X^0 are "elementary" particles and not reggeons.

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Article by V. Ya. Fainberg and L. V. Fil'kov, Vol. 5, No. 2.

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