[1] T. P. Das and A. K. Saha, Phys. Rev. 98, 516 (1955).
[2] T. P. Das and E. L. Hahn, Solid State Phys., Suppl. 1 (1958).

CONDUCTION ELECTRONS WITH SMALL EFFECTIVE MASSES

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In this note we consider several effects that occur for all metals whose electronic spectrum satisfies the following two requirements: (i) Several groups of conduction electrons belonging to different energy bands correspond to the same energy ϵ. (ii) A line of degeneracy points p_0, at which $\epsilon_1(p_0) = \epsilon_2(p_0)$, exists in quasimomentum space (1 and 2 denote the numbers of the bands, $\epsilon_{1,2}(p)$ are the dispersion laws).

The surfaces $\epsilon_{1,2}(p) = \epsilon$ corresponding to this type of spectrum are made up of two cavities belonging to different bands and having a single common point $p_0(\epsilon)$ $(\epsilon_1(p_0) = \epsilon_2(p_0) = \epsilon)$. The equal-energy surface near p_0 has the form of an elliptic cone. If the vector $\vec{\xi} = \vec{H}/H$ lies inside the solid angle made up by the normals to the cone surface, then the section of the equal-energy surface by the plane p_ξ = const is closed. At small values of the difference $|\delta p_\xi| = |p_\xi - p_{0\xi}(\epsilon)|$ $(p_{0\xi}(\xi) = p_0(\epsilon)\vec{\xi})$, the intersection area $S(\epsilon, p_\xi)$ and the effective mass $m^* = (2\pi)^{-1}(\partial S/\partial\epsilon)_{p_\xi}$, which determines the conduction-electron Larmor frequency $\Omega = eH/m^*c$, take the form:

$$S(\epsilon, p_\xi) = R(\vec{\xi})(p_\xi - p_{0\xi}(\epsilon))^2, \quad m^*(\epsilon, p_\xi) = R(\vec{\xi})(p_\xi - p_{0\xi}(\epsilon))/\pi v_\xi(\epsilon), \quad v_\xi^{-1} = \frac{dp_{0\xi}}{d\epsilon} \tag{1}$$

(the dimensionless constant $R(\vec{\xi}) = 1$, and the concrete form of R is immaterial; $v_\xi(\epsilon)$ is of the order of the characteristic velocity v_0). These formulas are due to the specific nature of the dispersion law near the degeneracy point:

$$\epsilon_{1,2}(p) = \epsilon_0(p_0) + A\delta p \pm \sqrt{(B\delta p)^2 + (C\delta p)^2}, \quad (A, B, C, \approx v_0, \ \delta p = p - p_0).$$

As seen from (1), $M^* \to 0$ linearly when $p_\xi \to p_{0\xi}$.

The vanishing of the effective mass is the basis for the effects considered in what follows.

Before proceeding to further investigations, we note first that the occurrence of the line of degeneracy points can be due to certain symmetry properties of the crystal lattice of the metal. This "forced" intersection of the bands occurs, in particular, for all metals which have a nontrivial screw axis or a glide plane [1]; examples are metals with close hexagonal packing (Zn, Mg, Cd, etc.). It can be shown that the degeneracy along the line is stable, i.e., it is not lifted at small lattice deformations. It follows therefore that the so-called "accidental degeneracy," which is not connected with the symmetry properties, is characteristic of many metals.

Since the kinematic mometum components p_ζ and p_η are not commuting operators in a magnetic field, the uncertainty principle ($\Delta p_\zeta \Delta p_\eta \gtrsim \sigma = (e\hbar H)/c$, $\vec{\zeta}$, $\vec{\eta} \perp \vec{\xi}$) precludes presence of stationary states for which $|\delta p_\xi| \ll \sigma^{1/2}$. When $\delta p_\xi \sim \sigma^{1/2}$ the classical approach is no longer valid, and a quantum analysis is needed for the investigation of the dynamics of an electron in a magnetic field. Using the method formulated by the author in [2,3] *), it can be shown that near the conical point the spectrum is determined by the well known quantization rules of I. Lifshitz and Onsager:

$$S(\epsilon, p_\xi) = \sigma(n + 1/2), \ n = 1,2... \tag{2}$$

$$\epsilon_n(p_\xi) = \epsilon_0(p_\xi) \pm v_0\sqrt{2\sigma(n+1/2)}. \tag{2a}$$

In (2a) it is assumed for simplicity that the cone is circular and that H is parallel to the cone axis.

According to (1), (2), and (2a), the distance between neighboring levels ($\Delta\epsilon_n = \epsilon_{n+1,p_\xi} - \epsilon_{n,p_\xi}$) increases for small δp_ξ, by a factor $\sqrt{\epsilon_0/\hbar\Omega_0} \gg 1$ over the "usual" distance (the order of which is $\hbar\Omega_0$), viz., $\Delta\epsilon_n \approx v_0\sigma^{1/2} = \sqrt{\epsilon_0\hbar\Omega_0}$ (Ω_0 and ϵ_0 are the characteristic frequency and energy). Anomalously large values of $\Delta\epsilon_n$ make resonance absorption of an alternating field with frequency $\omega \approx 10^{10} - 10^{11}$ sec^{-1} ($\omega\tau \gg 1$, τ = relaxation time) possible in very weak fields $H \gtrsim H^{(1)} = (\hbar c/e)(\omega^2/v_0^2) = 10^{-3} - 10^{-1}$ Oe **). In cyclotron resonance the magnetic field H, as usual, should be parallel to the surface of the metals. It follows from (2a) that the "resonant" values of H, corresponding to the minimum absorption, are determined by the equation

$$\hbar\omega = \epsilon_{m+n,p_\xi} - \epsilon_{n,p_\xi}, \quad H_{n,m} = \frac{c\hbar\omega^2}{2ev_0^2} / (\sqrt{m+n+1/2} - \sqrt{n+1/2})^2,$$

where n and m are integers and m is the resonance multiplicity. To observe quantum resonance oscillations it is necessary that the interval between resonant frequencies $\Omega_n = \Delta\epsilon_n/\hbar$ be larger than or of the order of $1/\tau$. In our case, when the frequency is $\omega \approx 10^{10} - 10^{11}$ cm^{-1} this requirement imposes an upper bound on H: $H \lesssim H^{(2)} = H^{(1)}\omega\tau \approx 10^{-2} - 10^{-1}$ Oe, i.e., $H^{(1)} \lesssim H \lesssim H^{(2)}$. As the magnetic field decreases from $H^{(2)}$ to $H^{(1)}$, the value of H and the amplitude of the hf field may become commensurate; in this case nonlinear effects set in and introduce additional irregularity in the dependence of the surface impedance Z on H.

Nonmonotonic behavior of the high-frequency impedance has by now been observed in a number of metals at fields $H \approx 1$ Oe [4]. However, these oscillations have not yet been unambiguously interpreted, since a similar dependence of Z on H may be due not only to quantum effects with small masses, but also to purely classical mechanisms. At any rate, our analysis shows that observation of quantum cyclotron resonance in very weak magnetic fields is perfectly realistic. (A more detailed quantitative study of resonance phenomena in weak fields will be the subject of a separate article).

In strong magnetic fields ($H \approx 10^3 - 10^4$ Oe) the existence of degeneracy points p_0 appears on the smooth part $\chi^{(sm.)}$ of the diamagnetic susceptibility of the metal. As is well known, $\chi^{(sm.)}$ is determined by the spectral region in which $S(\epsilon, p_\xi) = 0$ and is proportional to the characteristic Larmor frequency value. In this case we have $\Omega \to \infty$ when $S(\epsilon, p_\xi) = 0$, and this should lead to an appreciable growth of $\chi^{(sm.)}$. An exact analysis yields for $\chi^{(sm.)}$ the following formula when $T = 0$

$$\chi^{(sm.)} = \Lambda \frac{e}{c} \left(\frac{e\hbar}{cH}\right)^{1/2} \int \frac{dp_{o\xi}}{S''(p_{o\xi})} / (2\pi\hbar)^2 ; \quad \Lambda = \frac{3\pi}{2^{3/2}} \sum_{\ell=1}^{\infty} \frac{1}{\ell^{3/2}} ; \tag{4}$$

$$S'' = \left(\frac{d^2 S}{d\epsilon^2}\right)_{p_\xi} .$$

The integration in (4) is along the line of degeneracy points. As shown by (4), $\chi^{(sm.)}$ increases by a factor $\sqrt{\epsilon_0/\hbar\Omega_0} \approx 10^2$ compared with the usual case; attention is called to the appearance of an inverse-square-root dependence on H. Similar anomalies in the behavior of the smooth part of the diamagnetic susceptibility were observed recently in many metals [5].

The effects considered here can serve as one method of observing the degeneracy lines in electron spectra of metals. Another method is to investigate the interband magnetic breakdown that occurs when the magnetic field is situated outside the solid angle made up by the normals to the surface of the cone.

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[1] G. Ya. Lyubarskii, Teoriya grupp i ee primenenie v fizike (Group Theory and Its Use in Physics), Sec. 40, Gostekhizdat, 1957.

[2] A. A. Slutskin, JETP Letters 4, 96 (1966), transl. p. 65.

[3] A. A. Slutskin and A. M. Kadigrobov, Paper at LT-10, Moscow, September, 1966.

[4] M. S. Khaikin, JETP Letters 4, 164 (1966), transl. p. 113;
I. F. Koch and C. C. Kuo, Phys. Rev. 143, 470 (1965).

[5] V. I. Verkin, I. V. Svechkarev, and L. B. Kuz'micheva, Paper at LT-10, Moscow, September, 1966.

*) This method is closely related to the solution of the magnetic-breakdown problem

**) The values of H at which one can neglect the level smearing due to the finite lifetime of the quasiparticle is bounded from below by the condition $H \gg (\hbar c/e)/v_0^2\tau^2 \approx 10^{-6}$ Oe with $\tau \approx 10^{-9}$ sec).