can apparently be explained by the fact that, at least in the range of E, p, and f investigated by us, the relative decrease of ϵ with increasing f is determined only by the ratio of

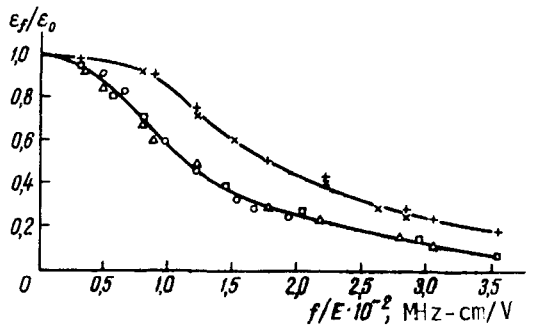

Fig. 2. ϵ_f/ϵ_0 vs. f/E: upper and lower curves are for E/p = 102 and 184 V/cm-mm Hg, respectively.

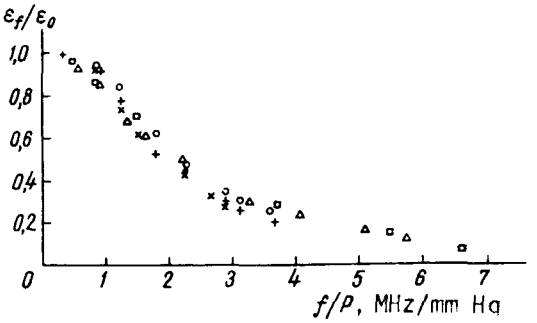

Fig. 3. ϵ_f/ϵ_0 vs. f/p; notation same as in Fig. 2.

the time of molecule precession in one direction to the time between molecule collisions. It can be assumed, however, that in the general case ϵ_f/ϵ_0 is determined by two ratios of the aforementioned frequencies.

We observed a similar influence of an alternating magnetic field on the thermal conductivity of oxygen. These results will be published.

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NONSTATIONARY SELF-FOCUSING OF LASER PULSES IN A DISSIPATIVE MEDIUM

S. A. Akhmanov and A. P. Sukhorukov
Physics Department, Moscow State University
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We discuss in this letter the influence of relaxation processes in a nonlinear medium on the dynamics of self-focusing of laser pulses [1,2]. Since the characteristic amplitude variation time is $\approx 10^{-9}$ sec even in "ordinary" giant pulses (see [3]; this time is even shorter in lasers with synchronized modes, $\approx 10^{-12}$ - 10^{-13} sec, see [4]), we cannot assume that the nonlinear response of the medium follows the field quasistatically. This pertains in particular to striction effects, which apparently determine the "fine" structure of self-focusing beams in liquids with large Kerr constants, and the "coarse" structures in such

liquids as CCl_4 and hexane (see [7]) and in solids. The dynamics of the refractive index of a medium in a specified light field were considered in [7,8]. At the same time, appreciable interest attaches to a study of the behavior of the light beam within times on the order of the relaxation times, i.e., a study of nonstationary self-focusing. The analysis of nonstationary self-focusing should obviously be based on a simultaneous solution of the field equations and the material equations of the medium. We present below such an analysis for striction and Kerr self-focusing of modulated waves, using a perturbation method that reveals the main features of the nonstationary process and permits an estimate of the characteristic times and the space scales. Principal attention is paid to the self-focusing length R_{nl}, the parameter easiest to determine experimentally. The formulas obtained for R_{nl} take into account both the damping of the light-beam field by linear and nonlinear absorption (these processes are important in the stationary theory, too), and the damping of the internal motions in liquids (swinging of molecules and of acoustic waves), which determine the relaxation time of the nonlinear polarization. The process of three-dimensional self-focusing of a quasiharmonic wave $E = A_0(t, z, r)\exp i[\omega t - kz - ks(t, z, r)]$ is described by the equations (see [9]):

$$\frac{1}{v}\frac{\partial s}{\partial t} + \frac{\partial s}{\partial z} + \frac{1}{2}\left(\frac{\partial s}{\partial r}\right)^2 = \frac{\epsilon_2'}{\epsilon_0'}p + \frac{1}{2k^2A_0}\left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right), \tag{1}$$

$$\frac{1}{v}\frac{\partial A_0^2}{\partial t} + \frac{\partial A_0^2}{\partial z} + \frac{\partial s}{\partial r}\frac{\partial A_0^2}{\partial r} + A_0^2\left(\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r}\right) + 2\delta_1 A_0^2 + 2\delta_2 A_0^4 = 0. \tag{2}$$

The quantity p, which is proportional to the polarizability of the medium, is described by the material equation

$$\tau\frac{\partial p}{\partial t} + p = A_0^2, \tag{3}$$

if the self-focusing is due to the Kerr effect, or by the wave equation

$$\left(-\Delta + \frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \frac{\Gamma}{v^2}\frac{\partial}{\partial t}\right)p = -\Delta A_0^2, \tag{4}$$

if the self-focusing is due to striction. For nonstationary effects connected with heating of the medium A_0^2 in (4) should be replaced by $\int_0^t A_0^2 dt$. Equations (1) and (2) have been written out for a medium in which $\epsilon = \epsilon_0 + \epsilon_2 E^2$; the stationary values of ϵ_0 and ϵ_2 are $\epsilon_0 = \epsilon_0' + i\epsilon_0''$ and $\epsilon_2 = \epsilon_2' + \epsilon_2''$; the linear response of the medium and the nonlinear loss are assumed to have no time lag. We present first some results concerning stationary self-focusing in a dissipative medium.

1. Stationary Self-focusing in a Dissipative Medium ($\frac{\partial}{\partial t} = 0$)

We calculate first the linear positive loss (passive medium). We consider perturbations in a plane damped wave

$$A_0^2 = E_0^2 \exp(-2\delta_1 z) + a\exp(-2\delta_1 z)\cdot \exp i(k\ r) \tag{5a}$$

and confine ourselves to the geometric-optics approximation ($k \to \infty$, the power exceeds the critical value). We then obtain for α

$$a = a_0 K_0^{-1}\left(\frac{1}{\delta_1 R^0_{n1}}\right) K_0[\exp(-\delta_1 z)/\delta_1 R^0_{n1}], \tag{5b}$$

where K_0 is a modified Bessel function, and $R^0_{n1} = a\sqrt{\epsilon_0'/2\epsilon_2' E_0^2}$ ($k_\perp = 1/a$) is the spatial scale of the stationary self-focusing in a lossless medium [9,10].

In the perturbation method this is the increment of a perturbation α that increases with the coordinate when $\delta_1 = 0$; $\alpha = \alpha_0 \exp(z/R^0_{n1})$. It is seen from (5) that the linear loss increases the self-focusing length. We can obtain an approximate estimate of the value of R_{n1} by assuming that the argument of the Bessel function changes by unity over this length:

$$a\sqrt{\epsilon_0'}/\sqrt{2\epsilon_2' E_0^2} \simeq \frac{1 - \exp(-\delta_1 R_{n1})}{\delta_1}. \tag{6}$$

The formulas presented agree well with the experiments described in [5] and with the semi-empirical relations derived there (the quantity $2\delta_1$ ranged in these experiments from 2×10^{-3} to 0.12 cm^{-1}). We can determine in similar fashion the self-focusing length in a medium with $\delta_1 < 0$ (active medium). We then have

$$a = a_0 I_0^{-1}(1/\delta R^0_{n1}) I_0[\exp(\delta_1 R^0_{n1})/\delta_1 R^0_{n1}]. \tag{7}$$

In this case $R_{n1} < R^0_{n1}$; the self-focusing of the beam proceeds more rapidly than in a lossless medium. To estimate the self-focusing, say, of the Stokes components of stimulated Raman scattering we can write $R_{n1}/z_\delta = [\ln(R^0_{n1}/z_\delta)]$ ($z_\delta = \delta_1^{-1}$), since here usually $R_{n1}/z_\delta \gg 1$. The same holds for semiconductor lasers, too.

In a medium with nonlinear absorption (we consider for simplicity the most typical case $\delta_1 \simeq 0$, $\delta_2 > 0$) we obtain for the perturbation of the amplitude profile $A_0^2 = E_0^2/(1 + \delta_2 E_0^2 z)$, with the aid of similar calculations,

$$R_{n1} = R^0_{n1} + (ka^2/2)\cdot(\epsilon_2''/\epsilon_2'). \tag{8}$$

The presence of nonlinear absorption increases the spatial scale of the self-focusing. R_{n1} does not vanish even when $E_0 \to \infty$, but tens to a constant value $R^{lim}_{n1} = (ka^2/2)(\epsilon_2''/\epsilon_2')$, which is determined by the diffraction length of the beam and by the nonlinear properties of the medium.

2. Nonstationary Processes in Self-focusing by the Kerr Effect

Substituting in (1), (2), and (3) a perturbed wave in the form

$$A_0^2 = E_0^2 + a\exp(i\nu t)\exp i\,\mathbf{k}_1\mathbf{r};\ s = s_0 + \phi;\ p = E_0^2 + p\exp(i\nu t) \tag{9}$$

and putting in these equations for simplicity $\delta_1 = \delta_2 = 0$ and $k \to \infty$, we arrive at an equation describing the amplitude perturbations

$$\frac{d^2a}{dz^2} - \frac{2\epsilon_2' E_0^2}{a^2\epsilon_0'(1+i\nu\tau)}\,a = 0, \tag{10}$$

hence

$$R_{nl} = R^0_{nl}\sqrt{2}\,\frac{\sqrt{1+(\nu\tau)^2}}{\sqrt{1+\sqrt{1+(\nu\tau)^2}}} \tag{11}$$

It follows from (11) that the self-focusing length of the modulated light wave increases in a relaxing medium. It is interesting that different spectral components of the envelope become self-focused in different cross sections, giving rise to a unique nonlinear "chromatic aberration" for the envelope. When $\nu\tau > 1$ we get $R_{nl} \simeq R^0_{nl}\sqrt{2\nu\tau}$, which should lead to a deformation of the envelope (which is noticeable for the pulses obtained in lasers with synchronized modes). The relaxation also has a noticeable effect on the behavior of plane quasimonochromatic waves; differences in the nonlinear response of the medium at different modulation frequencies lead in this case to "nonlinear dispersion." This effect can influence the limiting duration of the pulses passing through the nonlinear medium, and must be taken into account in the analysis of the temporal [6] and spatial-temporal [10] instability of a pulse in a nonlinear medium. Similar calculations can be made also with diffraction taken into account, in which case it turns out that the critical power is different for the different spectral components; in a relaxing medium, the critical power increases with increasing ν like $1 + (\nu\tau)^2$.

3. Nonstationary Processes in Striction Self-focusing

Substituting (9) in (1), (2), and (4) and assuming that $a(z) = \exp(\Lambda z)$, we get for Λ

$$\Lambda = \frac{1}{R^0_{nl}} \cdot \frac{1}{\sqrt{1 - \nu^2\tau^2 + i2\Gamma\nu\tau^2}}\,; \quad \tau_1 = \frac{a}{u}. \tag{12}$$

It follows from (12) that:

1. If the acoustic loss is equal to zero ($\Gamma = 0$), only the spectral components for which $\nu\tau_\perp \leq 1$ or $a/\lambda_{ac} < 1$ become self-focused ($\lambda_{ac} = u/\nu$).

2. When $\Gamma \neq 0$, self-focusing takes place for all the spectral components, but accompanied by strong nonlinear aberration. The minimum value of R_{nl} corresponds to $a/\lambda_{ac} = 1$ and is equal to $R^{min}_{nl} = R^0_{nl}\sqrt{2} \times \sqrt{\lambda_{ac}/Z_\delta}$, where $Z_\delta = u/2\Gamma$ is the "mean free path" of the phonon.

3. If $\nu\tau_\perp \gg 1$ and $\lambda_{ac}/Z_\delta < 1$ (a condition which is usually satisfied), we get $R_{nl} \simeq R^0_{nl}[(a/\lambda_{ac})(Z_\delta/\lambda_{ac})]$. We see from the latter formula that in this case a decrease in the loss increases the self-focusing length. The nonstationary behavior of striction self-focusing should become strongly manifest in solids, where Z_δ greatly exceeds the corresponding value for liquids. This pertains in particular to low-temperature experiments (see [11]); the phonon lifetimes reach here a value $\approx 10^{-4}$ sec. It must be emphasized that the results characterize the behavior of individual spectral components; to use them for an analysis of

self-focusing of pulses it is necessary to take into account the energy distribution over the spectrum. At the same time, these results are directly applicable to those cases when some frequency ν_0 is strongly pronounced in the spectrum of the envelope (for example, the frequency of intermode beats in a laser with giant pulse or the spike repetition frequency in a high-power laser operating in the free generation mode). Then the self-focusing beam can break up into "resonant" filaments with transverse scale $a_0 \simeq u/\nu_0$ (see [12]). We note that the foregoing procedure allows us to analyze temporal nonlinear aberrations connected with thermal effects; these aberrations are quite appreciable in the case of giant pulses.

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CONCERNING ONE POSSIBLE MECHANISM OF PRODUCTION OF THE MESIC-MOLECULAR ION $(dd\mu)^+$

E. A. Vesman
Institute of Physics and Astronomy, Estonian Academy of Sciences
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There is great disparity between the experimentally obtained rate of $(dd\mu)^+$ production [1] and the theoretical estimates [2]. It seems to us that this occurs because $(dd\mu)^+$ has a level $K = 1$, $\nu = 1$ with low binding energy. The binding energy obtained for this state in the adiabatic approximation [2] is $|E_d| \approx 7$ eV. Although variational calculations [3] did not yield this level, they do not, in our opinion, contradict its existence.