NEUTRINOS AND GRAVITONS IN THE ANISOTROPIC MODEL OF THE UNIVERSE

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Anisotropic cosmological solutions are widely discussed at the present time [1] in connection with the chemical composition of prestellar matter [2] and the primordial magnetic field [3]. These solutions approach asymptotically Friedmann's isotropic solution and in this respect they do not contradict the observations.

The picture of anisotropic expansion is radically altered when weakly-interacting particles (neutrinos and gravitons) are taken into account in the composition of prestellar matter.

The decrease in the anisotropy may be oscillatory. Nor can we exclude an appreciable increase in the present-day average energy of the relict neutrinos and gravitons compared with the isotropic model. This energy may exceed the present-day energy of relict electromagnetic quanta, corresponding to a temperature 3°K, and may even make the observation of neutrinos possible.

The reason for the singular behavior of weakly-interacting particles (we shall henceforth refer, for brevity, to neutrinos only) lies in the fact that in the absence of interaction the freely flying particles in an anisotropically expanding universe transform their momentum in such a way, that the momentum components vary differently in the directions of the different axes.

The equilibrium spherically-symmetrical momentum distribution of the particles changes into preferred motion along one axis (but with equal components in both directions).

As a result, the neutrino energy density decreases not as rapidly as in the anisotropic model filled with matter having isotropic pressure (we shall call the latter the Pascal model). The slower decrease in the energy density gives rise to an earlier gravitational influence (compared with the Pascal model) of the energy density on the expansion law itself (as is well known [4], in the limit as $\rho \to \infty$ the presence of matter has no effect on the deformation law).

On the other hand, if the anisotropy is accompanied by compression along one of the axes during the early stage, some of the neutrinos and antineutrinos acquire such an energy, that the probability of their irreversible transformation into electrons and positrons becomes noticeable. This leads to an increase in the entropy of the medium. (The formulas are given at the end of this note.) Owing to such "heating," the energy density of the quanta and of the pairs decreases more slowly than in the Pascal model, and has the same order of magnitude as the neutrino energy density. *

Starting with some instant τ, the energy density already exerts an effect on the expansion. If by that instant $\epsilon_\nu \gg \epsilon_{\gamma,e^\pm}$, then the cosmological model is described by a solution in which the energy density is determined only by neutrinos moving along one axis.

Such a solution is presented at the end of this note. The main property of the solution is rapid expansion in the direction of neutrino motion and slow expansion in the transverse directions. As a result, the neutrino energy is rapidly decreased along the axis because of the red shift, and the principal role in the energy density is assumed by the quanta and pairs. The solution rapidly becomes isotropic. The isotropization time is of the order of τ. However, the neutrino momentum distribution remains strongly anisotropic. Disregarding the influence of other weakly-interacting particles, the energy density of the neutrinos and of the γ quanta in the anisotropic model should at the present time be of the same order of magnitude, but the neutrino flux is strongly anisotropic and the average neutrino energy is much higher than the γ-quantum energy.

In the special case of very small α (see the formulas at the end of the article), the "heating" by the anisotropic neutrinos is insignificant, the approach to the isotropic solution may be oscillatory, the present-day distribution of the neutrino momenta is isotropic, and the neutrino energy is of the order of the γ-quantum energy.

We did not take into account other weakly-interacting particles (such as gravitons), which may interact with one another and with the remaining particles in a manner different than the neutrinos. These particles may give rise to strong and prolonged oscillations of the anisotropy, survival of a large number of high-energy particles to the present time, and other phenomena. Instability of the directional neutrino flux and the flux of other free particles, leading to scattering, is also possible.

A complete analysis of all possible variants (in particular, with inclusion of the magnetic field), is quite difficult and has not yet been carried out completely. At the same time, it is clear even now that unless account is taken of the role of weakly-interacting particles, it is impossible to consider the theory of the anisotropic universe, and in particular the question of the chemical composition of prestellar matter.

In conclusion, we present a few formulas.

Near the singularity, the solution does not depend on the presence of matter, and takes for the homogeneous plane anisotropic model the form [5]:

$$ds^2 = c^2dt^2 - t^{2p_1}dx_1^2 - t^{2p_2}dx_2^2 - t^{2p_3}dx_3^2 . \tag{1}$$

We put $\alpha = -\min(p_1; p_2; p_3)$; $0 < \alpha < 1/3$ [4]. The energy density in the Pascal stage (prior to the separation of the neutrino) changes like $\epsilon = kt^{-4/3}$. In Friedmann's isotropic model, the neutrinos are separated at the instant $t \equiv \tau \simeq 10^{-1}$ sec [6]. The instant of neutrino separation in model 1 is expressed in terms of the constant $\theta = (\kappa k)^{-3/2}$, namely $t_0 \simeq \theta(r/\theta)^{9/4}$ (κ - Einstein's gravitational constant). The separation is followed by a non-Pascal stage. During this stage the entropy growth is given by

$$S/S_0 = \{1 + \frac{4\alpha}{1-\alpha}[(t/t_0)^{(1-\alpha)/3} - 1]\}^{3/4} \tag{2}$$

For $t \gg t_0$ the ratio of the neutrino energy density to that of the γ quanta and pairs is $\epsilon_{\gamma e\pm}/\epsilon_\nu \simeq [4\alpha/(1-\alpha)] + (t_0/t)^{(1-\alpha)/3}$. The instant τ, when the influence of the energy density on the solution begins and the entropy growth ends is expressed in terms of the constant θ:

$$\tau \simeq \theta\left(\frac{r}{\theta}\right)^{(9/4)[(1-\alpha)(3-\alpha)]} \qquad (3)$$

when $\theta > r$ and we have simply $\tau = \theta$ and no entropy growth when $\theta < r$. When the instant τ is reached (for small values of α, $\alpha > (t_0/t)^{(1-\alpha)/3}$), the density ratio is $(n_\nu/n_{\gamma e\pm}) \simeq (r/\theta)^{(3/8)[(3+5\alpha)(3-\alpha)]}$, and the ratio of the average particle energies is

$$\frac{E_\nu}{E_\gamma} \simeq \frac{n_\gamma}{n_\nu} \simeq \left(\frac{r}{\theta}\right)^{(-3/8)[(3+5\alpha)(3-\alpha)]}.$$

For $t > \tau$ we have

$$S(t) \simeq S(\tau);\ (n_\nu/n_\gamma) \simeq (n_\nu/n_\gamma)|_{t=\tau};\ (E_\nu/E_\gamma) \simeq (E_\nu/E_\gamma)|_{t=\tau}$$

The cosmologic solution itself, with a neutrino moving along the x_3 axis, and with $T_3^3 = -T_0^0 = \epsilon$ and $T_1^1 = T_2^2 = 0$, takes the form

$$ds^2 = c^2dt^2 - a_1^2dx_1^2 - a_2^2dx_2^2 - a_3^2dx_3^2;$$

$$a_1 = a_{10}y^{1/2+p_1};\ a_2 = a_{20}y^{1/2-p};\ a_3 = a_{30}y^{p^2-1/4}e^{y};\ -1/2 \leqslant p \leqslant 1/2;$$
$$t = \tau_0\int y^{p^2-1/4}e^{y}dy;\ \kappa\epsilon = \tau_0^{-2}y^{-1/2}e^{-2y};\ 0 < y < \infty.$$

The instant τ corresponds to $y = 1/4 - p^2$. When $t \gg \tau$, the neutrino energy decreases like $\epsilon \approx t^{-2}$.

[1] K. S. Thorne, Preprint CST, California, 1966.

[2] S. W. Hawking and R. J. Tayler, Nature 209, 1278 (1966).

[3] Ya. B. Zel'dovich, JETP 48, 986 (1965), Soviet Phys. JETP 21, 656 (1965).

[4] E. M. Lifshitz and I. M. Khalatnikov, UFN 80, 391 (1963), Soviet Phys. Uspekhi 6, 495 (1964).

[5] E. Schücking and O. Heckmann, 11 Conseil de Physique, Solvay, Bruxelles, 1958.

[6] Ya. B. Zel'dovich, UFN 89, 647 (1966), Soviet Phys. Uspekhi 9 (1967), in press.

* Provided the exponent of the time in the dependence of the distance on the time is not especially small on the axis along which the compression takes place (see the end of the note). This limitation is implied throughout.