INSTABILITY, CONNECTED WITH THE "LOSS CONE" OF A PLASMA WITH HOT ELECTRONS

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The instability connected with the "loss cone" in a plasma with hot ions and cold electrons was first considered in [1]. It is the consequence of the anisotropy of the ion-velocity distribution function, since there are no ions in the "loss cone." A similar instability can arise in a plasma placed in a magnetic mirror trap and consisting of hot electrons whose velocity distribution function is zero in the loss cone and is much higher than that of a cold plasma with isotropic distribution function. Such a plasma was produced in a number of experiments (see, for example, [2,3]).

Let the hot-electron distribution function be $n'f(v_{\parallel}, v_{\perp}) = 0$ at $|v_{\parallel}| > \alpha v_{\perp}$, $\alpha = \sqrt{R - 1}$, R - mirror ratio, n' - hot-electron density, i.e., $\int f\, dv = 1$. The cold plasma has a density n_0 and a temperature T_0, $T_0/m \ll \bar{v}^2 = \int v^2 f\, dv$. Then, provided the conditions $k_{\perp} \gg k_{\parallel}$, $\gamma = \mathrm{Im}\,\omega \geq \omega_H$ or $k_{\parallel}\bar{v} \gtrsim \omega_H$, $\omega_H \ll \omega_0$, $\omega \gg \omega_H$ ($\omega_H = eH/mc$, $\omega_0 = \sqrt{[4\pi e^2(n_0 + n')]/m}$ - plasma frequency, and ω - oscillation frequency) are satisfied, the following dispersion equation can be obtained

$$1 - \frac{\omega_{0i}^2}{\omega^2} - \frac{\omega_0^2}{\omega^2}\frac{n_0}{(n_0+n')}\left(1 - \frac{i\nu}{\omega}\right) + \frac{\omega_0^2}{k^2\bar{v}^2}\frac{n'}{(n_0+n')}\left\{\int_0^{\omega/k\bar{v}} \frac{|\omega|}{k\bar{v}}\frac{(\partial\psi/\partial x)\,dx}{\sqrt{\dfrac{\omega^2}{k^2\bar{v}^2} - x^2}} - i\int_{\omega/k\bar{v}}^{\infty}\frac{\omega}{k\bar{v}}\frac{(\partial\psi/\partial x)\,dx}{\sqrt{x^2 - \dfrac{\omega^2}{k^2\bar{v}^2}}}\right\} = 0 \qquad (1)$$

where

$$x = \frac{v_{\perp}}{\bar{v}}; \quad \psi = 2\pi\bar{v}^2\int f_0\, dv_{\parallel}; \quad \int_0^{\infty} x\,dx\,\psi = 1; \quad \omega_{0i}^2 = \frac{4\pi e^2(n_0 + n')}{M}$$

ν - frequency of electron-ion collisions in the cold plasma. The term with ν takes into account the friction between the cold electrons and the ions.

Let us consider first the case when $n' \ll n_0$. We then find from (1) that

$$\omega = \omega_0 - \frac{i\nu}{2} + \frac{i\omega_0}{2}\frac{\omega_0^2}{k^2\bar{v}^2}\frac{n'}{n_0}\phi\left(\frac{\omega_0}{k\bar{v}}\right), \qquad (2)$$

$$\phi = -\int_{\omega/k\bar{v}}^{\infty}\frac{\omega}{k\bar{v}}\frac{\partial\psi/\partial x}{\sqrt{x^2 - \omega^2/k^2\bar{v}^2}}\,dx.$$

The function $\phi(\omega_0/k\bar{v})$ is positive when $(\omega_0/k\bar{v} \lesssim 1$, $|\phi(\omega_0/k\bar{v})| \lesssim 1$, i.e., the unstable waves are those with $k > (\omega_0/\bar{v})$. Electron-ion collisions can suppress the instability. The instability vanishes if

$$n' < n_0 \frac{\nu}{\omega_0} \frac{1}{\mu}, \tag{3}$$

where

$$\mu = \max\left(\frac{\omega_0^2}{k^2 \bar{v}^2} \phi\left(\frac{\omega_0}{k\bar{v}}\right)\right).$$

Using for ν the expression (4)

$$\nu = \frac{8\pi n_0 \Lambda e^4}{3\sqrt{3m}T_0^{3/2}} = \frac{8\pi n e^4 \Lambda}{m^2 c_0^3},$$

where Λ is the Coulomb logarithm, we find that in order to stabilize the instability we must satisfy the condition (T_0 is in electron volts)

$$n' < \frac{\omega_0^3 \Lambda}{c_0^3 \mu} \approx 0{,}7 \cdot 10^{-10} \frac{\Lambda}{\mu} \left(\frac{n_0}{T_0}\right)^{3/2}. \tag{4}$$

The value of μ and the form of the function $\phi(\omega_0/k\bar{v})$ is determined by the concrete form of the function f_0 and consequently depends on the mirror ratio. To estimate this dependence, we choose the model function

$$f_{0M} = \frac{\sqrt{1+\alpha^2}}{\alpha} \frac{1}{\pi^{3/2}\bar{v}^3} \exp\left[-\frac{1}{\bar{v}^2}(v_\perp^2 + v_\parallel^2)\right] \qquad |v_\parallel| > \alpha v_\perp$$
$$f_{0M} = 0 \qquad |v_\parallel| > \alpha v_\perp \tag{5}$$

Actually, f_0 will not be cut off abruptly at $|v_\parallel| = \alpha v_\perp$. This, however, has little effect on the form of $\phi(\omega/k\bar{v})$ (see [5], where numerical calculation results confirming this fact are given). For $f_0 = f_{0M}$, the expression in the curly brackets of (1) takes the form

$$F\left(\frac{\omega}{k\bar{v}}\right) = 2\frac{\omega}{k\bar{v}} \frac{\sqrt{1+\alpha^2}}{\alpha} \Big\{ \sqrt{\pi} \Big[\alpha \exp\left[-\frac{(1+\alpha^2)}{2} \frac{\omega^2}{k^2\bar{v}^2}\right] I_0\left(\frac{(1+\alpha^2)}{2} \frac{\omega^2}{k^2\bar{v}^2}\right) -$$
$$- \exp\left(-\frac{\omega^2}{k^2\bar{v}^2}\right) \alpha \int_0^{\omega^2/k^2\bar{v}^2} dx \exp\left[-x\frac{(3+\alpha^2)}{2}\right] I_0\left(\frac{1+\alpha^2}{2}x\right)\Big] - \frac{i}{\sqrt{\pi}}\Big[\alpha \exp\left[-\frac{(1+\alpha^2)}{2} \times\right.$$
$$\times \frac{\omega^2}{k^2\bar{v}^2}\Big[K_0\left(\frac{(1+\alpha)^2}{2} \frac{\omega^2}{k^2\bar{v}^2}\right) - \exp\left(-\frac{\omega^2}{k^2\bar{v}^2}\right) \alpha \int_0^{\omega^2/k^2\bar{v}^2} dx \exp\left[x\frac{(1-\alpha^2)}{2}\right] K_0\left(\frac{1+\alpha^2}{2}x\right) - \tag{6}$$
$$- 2\exp\left(-\frac{\omega^2}{k^2\bar{v}^2}\right) \arcsin\frac{\alpha}{\sqrt{1+\alpha^2}}\Big]\Big\},$$

where I_0 and K_0 are Bessel functions of imaginary argument. Figures 1 and 2 show plots of

μ, y_0, y_m, and $\mathrm{Re}F(y_0, \alpha)$ against the mirror ratio R, obtained from (6) by numerical calculation (the value of y_0 was determined from the equation $\mathrm{Im}F(y_0) = 0$, and the value of y_m corresponds to the phase velocity at which the instability increment, defined by (2), is maximal).

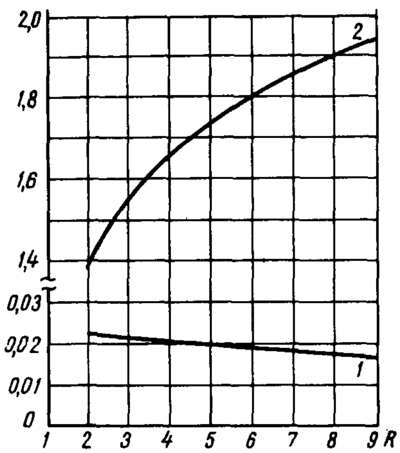

Fig. 1. Plot of μ (curve 1) and of $\mathrm{Re}F(y_0)$ (curve 2) vs. the mirror ratio R.

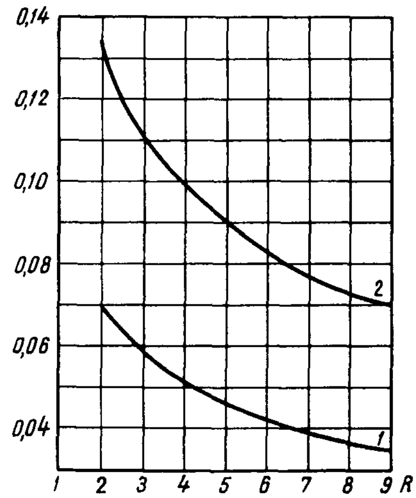

Fig. 2. Plot of y_m (curve 1) and y_0 (curve 2) vs. the mirror ratio R.

We see from the figures that for the mirror ratios R = 2 - 5 usually encountered in experiment, and for a value of the Coulomb logarithm $\Lambda \approx 20$, the instability vanishes if (T_0 is in electron volts)

$$n' \lesssim 7 \cdot 10^{-8} \left(\frac{n_0}{T_0}\right)^{3/2}. \tag{7}$$

We now consider the case when $n_0 \lesssim n'$, and take into account the finite nature of the temperature of the cold electron component T_0. Using the Nyquist criterion, we find that the instability will develop if the following inequality is satisfied

$$n_0 \gtrsim 10 n' R^{1/2} \left(\frac{T_0}{m\bar{v}^2}\right)^{3/2}, \tag{8}$$

i.e., if the concentration of the cold electrons exceeds a certain critical value. Condition (8) takes into account the Landau damping of the unstable oscillations by the cold electrons.

We can consider similarly the case when there are no cold electrons, and their role is assumed by cold ions. To this end it is necessary to substitute m/M for n_0/n' and T_i (the ion temperature) for T_0 in the inequality (8). We find that instability takes place if

$$\frac{T_i}{m\bar{v}^2} \lesssim \left(\frac{m}{M}\right)^{2/3} \tag{9}$$

Thus, a strong difference between the electron and ion temperatures is necessary in order for the instability to develop.

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MEASUREMENT OF THE SHUBNIKOV - DE HAAS EFFECT IN GRAPHITE AT PRESSURES UP TO 8 KBAR

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The electronic spectrum of graphite is usually described by the Slonczewski-Weiss model (SW). In this model, the dependence of the energy on the quasimomentum is expressed on the basis of general principles [1] in terms of parameters characterizing the conduction-electron interactions. The Brillouin zone for graphite is a hexagonal prism with a base-side length $2\pi/a_0$ and a height $2\pi/c_0$ (a_0 = 2.46 Å and c_0 = 4.70 Å). The Fermi surface of graphite occupies a small section of the Brillouin zone and is located along the vertical edges of the zone. In the SW model we can write for the dispersion law the simplified expressions:

$$E_{\pm} = 2\gamma_2 \cos^2\phi \pm \hbar^2\kappa^2/2m^*(\phi), \quad (1)$$

$$m^*(\phi) = {}^4/{}_3 (\hbar/a_0)^2 (\gamma_1/\gamma_0^2) \cos\phi, \quad (2)$$

where $\phi = k_z c_0/2$, κ is reckoned from the edge of the Brillouin zone, γ_0 is the interaction energy of the conduction electrons in the plane of the layer, γ_1 the interaction energy of the conduction electrons of the neighboring layers, γ_2 is the same energy through the layer, and the + and - signs correspond to electrons and holes.

From this, recognizing that the Fermi energy $\epsilon_F = (4/3)\gamma_2$, we can readily obtain the value of the extremal section of the hole part of the Fermi surface of graphite, parallel to the (0001) plane,

$$S^h_{extr} = (4/3)^2 \pi/a_0^2(\gamma_1\gamma_2/\gamma_0^2) = 0{,}923\gamma_1\gamma_2/\gamma_0^2 \text{ Å}^{-2}. \quad (3)$$

The expression for the analogous section of the electron surface differs only in the value of the coefficient preceding $\gamma_1\gamma_2/\gamma_0^2$. The SW model was experimentally confirmed in a number of recent papers [2-3].