RELATIONS BETWEEN THE INTERACTION CONSTANTS OF σ, f, ρ, AND ω MESONS

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We consider in this paper a method for obtaining equations for particle interaction constants. Unlike the usual bootstrap equations [1], which are obtained by imposing definite conditions on the reaction amplitude at the resonance point, we find the equations in this paper by equating two dispersion relations (d.r.) for the amplitude of the investigated process, one of which is written in terms of t and s with fixed u, and the other in terms of s and u with fixed t. The method is considered using as an example elastic scattering of γ quanta by pions, and is used to determine the interaction constants of σ and f mesons.

We denote by k_1, q_1, k_2, and q_2 the 4-momenta of the initial and final photons and pions, and introduce the invariant variables $s = (q_1 + k_1)^2$, $u = (q_1 - k_2)^2$, and $t = (k_2 - k_1)^2$. We represent the amplitude for the scattering of γ quanta by pions with the aid of a complete system of orthogonal vectors [2] in the form

$$T = \frac{(E_2 P')\,(E_1 P')}{(P')^2}\, T_1(s,u,t) + \frac{(E_2 N)(E_1 N)}{N^2}\, T_2(s,u,t)\,. \tag{1}$$

We assume that the amplitude difference $T_i(s, t) - T_i(s, t = 0)$ tends to a constant as $s \to \infty$ (and for $t < 0$), and increases more slowly than t when $t \to \infty$. We write for this difference, on the one hand, the d.r. in terms of t and s with fixed u, and on the other the subtractionless closed-loop d.r. in terms of s and u with fixed t. Equating these relations we obtain

$$\begin{aligned}\frac{t}{\pi}\int_{4\mu^2}^{\infty}\frac{A_{3i}(t',u)}{t'(t'-t)}\,dt' &= \frac{t}{\pi}\int_{4\mu^2}^{\infty}\frac{A_{1i}(s',u)-A_{1i}(s',t=0)}{(s'-s)(s'-s-t)}\,ds' + \\ &+\frac{1}{\pi}\int_L ds'\,[A_{1i}(s',t)-A_{1i}(s',t=0)]\left(\frac{1}{s'-s}+\frac{1}{s'-u}\right)+\phi_i(s,t)-\phi_i(s,t=0),\end{aligned} \tag{2}$$

where A_{3i} and A_{1i} are the imaginary parts of the amplitudes in the t and s channels respectively, the second integral in the right side of (2) is taken along a segment L along the real axis, and $\varphi_i(s, t)$ are integrals over semicircles. We assume now that the difference $A_{1i}(s, t) - A_{1i}(s, t = 0)$ decreases sufficiently rapidly when $s \to \infty$ and $t < 0$, so that the integral of this difference in (2) is meaningful when $L \to \infty$. Under the foregoing assumption we have when $L \to \infty$

$$\phi_i(t) - \phi_i(t=0) = \frac{t}{\pi}\int_{4\mu^2}^{\infty}\frac{A_{3i}(t')}{t'(t'-t)}\,dt'\,, \tag{3}$$

where $A_{3i}(t)$ is that part of the amplitude $A_{3i}(t', u)$ which depends only on t. Substituting

(3) in (2) and letting L go to infinity, we obtain the following equation at the point $t = 0$, $s = u = \mu^2$

$$\int_{4\mu^2}^{\infty} \frac{\Phi_{3i}(t', u=\mu^2)}{t'^2}\, dt' = \int_{4\mu^2}^{\infty} ds' \left\{ \frac{A_{1i}(s', u=\mu^2) - A_{1i}(s', t=0)}{(s'-\mu^2)^2} + \right.$$

$$\left. + \frac{2}{s'-\mu^2} \frac{\partial}{\partial t} A_{1i}(s', t)\Big|_{t=0} \right\}. \tag{4}$$

In this equation $\Phi_{3i} = A_{3i}(t', u) - A_{3i}(t')$.

Let us consider the amplitudes T_i of γ-π scattering in a state with isotopic spin of two pions $I = 0$. The main contribution to the integral of $\Phi_{3i}^{(0)}$ will be made in this case by the scalar meson (σ) and by the f meson. It must be assumed here, that the σ and f mesons are Regge poles and that the corresponding $a(0) < 0$. Assuming further that the main contribution to the right side of (4) is made by ρ and ω mesons, we obtain two equations relating the interaction constants of the σ, f, ρ, and ω mesons

$$\begin{cases} \dfrac{\Lambda_{\sigma\to2\gamma}\, g_{\sigma\pi\pi}}{\mu_\sigma^4} + \dfrac{\Lambda_{f\to2\gamma}\, g_{f\pi\pi}\, \mu}{2\mu_f^2} = 0 \\ 3\left(\dfrac{\Lambda_{\sigma\to2\gamma}\, g_{\sigma\pi\pi}}{\mu_\sigma^4} - \dfrac{\Lambda_{f\to2\gamma}\, g_{f\pi\pi}\, \mu}{2\mu_f^2}\right) = -\dfrac{8\mu_0^2}{(\mu_0^2-\mu^2)^2}\left(g_{\omega\to\pi\gamma}^2 + 3g_{\rho\to\pi\gamma}\right) \end{cases} \tag{5}$$

where μ_σ, μ_f, μ_ρ, and μ_ω are the respective masses of the σ, f, ρ, and ω mesons, and where we put for simplicity $\mu_\rho \cong \mu_\omega \equiv \mu_0$. Taking into account the connection of $\Lambda_{\sigma\to2\gamma}$, $\Lambda_{f\to2\gamma}$, $g_{\sigma\pi\pi}$, $g_{f\pi\pi}$, $g_{\omega\to\pi\gamma}$, and $g_{\rho\to\pi\gamma}$ with the widths of the corresponding decays, we obtain from (5)

$$\sqrt{\Gamma_{f\to2\gamma}\Gamma_{f\to2\pi}} \approx \frac{\mu_f^5}{10\sqrt{3}\,\mu^2\mu_0^3}\left[\Gamma_{\omega\to\pi\gamma} + 3\Gamma_{\rho\to\pi\gamma}\right], \tag{6}$$

$$\sqrt{\Gamma_{\sigma\to2\gamma}\Gamma_{\sigma\to2\pi}} = -\left(\frac{\mu_\sigma}{\mu_0}\right)^3\left[\frac{(\mu_\sigma^2-4\mu^2)^{1/2}}{2\mu_\sigma}\right]^{1/2}\left(\Gamma_{\omega\to\pi\gamma} + 3\Gamma_{\rho\to\pi\gamma}\right). \tag{7}$$

Putting $\Gamma_{\omega\to\pi\gamma} = 1.08$ MeV [3] and $\Gamma_{\rho\to\pi\gamma} = 0.1$ Mev, we obtain from (6)

$$\sqrt{\Gamma_{f\to2\gamma}\Gamma_{\rho\to2\pi}} \simeq 28{,}6 \text{ MeV} \tag{8}$$

It is known that the f meson decays essentially into two pions. If we take for the total width of the f meson decay $\Gamma_f = 120$ MeV and assume that $\Gamma_{f\to2\pi}/\Gamma_f = 0.9$, we obtain $\Gamma_{f\to2\gamma}/\Gamma_f \approx 0.0063$.

Let us consider the σ meson. We take for the mass and total decay width of the σ meson [4] $\mu_\sigma = 720$ MeV and $\Gamma_\sigma = 50$ MeV. For the given value of μ_σ we get from (7) $\Gamma_{\sigma\to2\gamma}\Gamma_{\sigma\to2\pi} \approx 0.549$ MeV^2. Assuming that the main decay of the σ meson is into two pions and that $\Gamma_{\sigma\to2\pi}/\Gamma_\sigma = 0.9$, we get $\Gamma_{\sigma\to2\gamma}/\Gamma_\sigma \simeq 2.44 \times 10^{-4}$.

For amplitudes with two-pion isotopic spin I = 2, equation (4) yields

$$\int_{4\mu^2}^{\infty} \frac{\Phi_{31}^{(2)}(t', u = \mu^2)}{t'^2}\, dt' = 0, \tag{9}$$

$$\int_{4\mu^2}^{\infty} \frac{\Phi_{32}^{(2)}(t', u = \mu^2)}{t'^2}\, dt' = \sqrt{\frac{2}{3}}\, \frac{8\mu_\omega^2}{\mu_\omega^2 - \mu^2}\, g^2_{\omega\to\gamma\pi} \approx \sqrt{\frac{2}{3}}\, \frac{48\pi}{\mu_\omega^3}\, \Gamma_{\omega\to\pi\gamma}\ .$$

It follows from (9) and (5) that the contribution of the states with I = 2 to the γ-π scattering is comparable with the contribution from the states with I = 0. Since the right sides of the equations for the amplitudes $\Phi_{31}^{(0)}$ and $\Phi_{32}^{(0)}$ are similar to those for $\Phi_{31}^{(2)}$ and $\Phi_{32}^{(2)}$, and furthermore the main contribution to $\Phi_{31}^{(0)}$ is made by states with total spins I = 0 and I = 2, we can expect a strong interaction in the states with I = 0 and I = 2 also in the case when I = 2.

It should be noted that if experiment confirms the correctness of the derived relations, this will serve as evidence in favor of the premise that the Regge poles of the t-channel with $\alpha(0) \geq 0$ make no contribution to γ-π scattering, while the $\alpha_j(0)$ corresponding to σ and f mesons are negative.

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MAGNETIC UNIVERSE WITH MATTER

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By electromagnetic universe is meant the simultaneous solution of Maxwell's equations for the electromagnetic field and Einstein's equations for the gravitational field. The properties of the electromagnetic universe were discussed recently in several papers [1,2].

Of great astrophysical interest is the so-called cylindrical magnetic universe. In this stationary solution of the gravitational equations in vacuum, the magnetic field with cylindrical symmetry is determined completely by the metric of the space, and the metric is determined in turn by the energy-momentum tensor of the magnetic field. Such a solution in vacuum (zero electric current), as shown by Melvin [1], is stable against small perturbations.