For amplitudes with two-pion isotopic spin I = 2, equation (4) yields

$$\int_{4\mu^2}^{\infty} \frac{\Phi_{31}^{(2)}(t', u = \mu^2)}{t'^2}\, dt' = 0,$$

$$\int_{4\mu^2}^{\infty} \frac{\Phi_{32}^{(2)}(t', u = \mu^2)}{t'^2}\, dt' = \sqrt{\frac{2}{3}}\, \frac{8\mu_\omega^2}{\mu_\omega^2 - \mu^2}\, g^2_{\omega\to\gamma\pi} \approx \sqrt{\frac{2}{3}}\, \frac{48\pi}{\mu_\omega^3}\, \Gamma_{\omega\to\pi\gamma}\,. \qquad (9)$$

It follows from (9) and (5) that the contribution of the states with I = 2 to the γ-π scattering is comparable with the contribution from the states with I = 0. Since the right sides of the equations for the amplitudes $\Phi_{31}^{(0)}$ and $\Phi_{32}^{(0)}$ are similar to those for $\Phi_{31}^{(2)}$ and $\Phi_{32}^{(2)}$, and furthermore the main contribution to $\Phi_{31}^{(0)}$ is made by states with total spins I = 0 and I = 2, we can expect a strong interaction in the states with I = 0 and I = 2 also in the case when I = 2.

It should be noted that if experiment confirms the correctness of the derived relations, this will serve as evidence in favor of the premise that the Regge poles of the t-channel with $\alpha(0) \geq 0$ make no contribution to γ-π scattering, while the $\alpha_j(0)$ corresponding to σ and f mesons are negative.

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MAGNETIC UNIVERSE WITH MATTER

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By electromagnetic universe is meant the simultaneous solution of Maxwell's equations for the electromagnetic field and Einstein's equations for the gravitational field. The properties of the electromagnetic universe were discussed recently in several papers [1,2].

Of great astrophysical interest is the so-called cylindrical magnetic universe. In this stationary solution of the gravitational equations in vacuum, the magnetic field with cylindrical symmetry is determined completely by the metric of the space, and the metric is determined in turn by the energy-momentum tensor of the magnetic field. Such a solution in vacuum (zero electric current), as shown by Melvin [1], is stable against small perturbations.

K. Thorne [2] has also demonstrated the absolute stability against arbitrary perturbations which do not violate the cylindrical symmetry.

We consider here solutions of the magnetic-universe type in the presence of uncharged matter described by an equation of state $p = \alpha\epsilon$. We shall seek a solution in a co-moving reference frame ($u_0 u^0 = -1$, $u_\alpha = 0$) with diagonal metric that depends on one variable (x^2)

$$-ds^2 = \nu_i e^{2F_i} (dx^i)^2 , \quad F_i = F_i(x^2) \quad (\nu_0 = -1, \nu_\alpha = 1). \tag{1}$$

The magnetic field is directed along the x^3 axis. Then integration of Maxwell's equation

$$\frac{\sigma F^{ik}\sqrt{-g}}{\sigma x^k} = 0 \tag{2}$$

yields

$$F'^{\,2} = \frac{H_o}{\sqrt{-g}} = \frac{H_o}{e^{F_0+F_1+F_2+F_3}} , \tag{3}$$

where H_0 is a certain characteristic constant magnetic field. The nonvanishing components of the energy-momentum tensor of the electromagnetic field ($T_i^k = \frac{1}{4}\pi(F_{ie}F^{k\ell} - \frac{1}{4}\delta_i^k F_{\ell m}F^{\ell m})$) is equal to

$$T_1^1 = T_2^2 = -T_3^3 = -T_4^4 = \frac{H_0^2}{8\pi} \frac{e^{2F_1}}{e^{2(F_0+F_1+F_3)}} , \tag{4}$$

and that of the energy-momentum tensor of the matter ($T_i^k = (\epsilon + p)u_i u^k + p\delta_i^k$) is

$$T_1^1 = T_2^2 = T_3^3 = a\epsilon, \quad T_0^0 = -\epsilon , \quad T = (-1 + 3a)\,\epsilon. \tag{5}$$

We can now write the corresponding components of Einstein's equations

$$(R_i^k = \frac{8\pi\kappa}{c^4}(T_i^k - \frac{1}{2}\delta_i^k T))$$

$$R_o^o = e^{-2F_2}[-F_o'' + F_o'(F_2' - F_o' - F_1' \cdot F_3')] = \frac{8\pi\kappa}{c^4}[-\frac{H_o^2}{8\pi}\frac{e^{2F_1}}{e^{2(F_o+F_1+F_3)}} - $$
$$- \frac{1}{2}(1 + 3a)\,\epsilon], \tag{6}$$

$$R_1^1 = e^{-2F_2}[-F_1'' + F_1'(F_2' - F_1' F_3')] = \frac{8\pi\kappa}{c^4}[\frac{H_o^2}{8\pi}\frac{e^{2F_1}}{e^{2(F_o+F_1+F_3)}} +$$
$$+ \frac{1}{2}(1 - a)\,\epsilon], \tag{7}$$

$$R_3^3 = e^{-2F_2}[-F_3'' + F_3'(F_2' - F_o' - F_1' - F_3')] = \frac{8\pi\kappa}{c^4}[-\frac{H_o^2}{8\pi}\frac{e^{2F_1}}{e^{2(F_o+F_1+F_3)}} +$$
$$+ \frac{1}{2}(1 - a)\,\epsilon], \tag{8}$$

$$R^2_2 = e^{-2F_2}[-(F''_0+F''_1+F''_3) + F'_2(F'_0+F'_1+F'_3) - (F_0'^2 + F_1'^2 + F_3'^2)] =$$

$$\frac{8\pi\kappa}{c^4}\left[\frac{H_0^2}{8\pi}\frac{e^{2F_1}}{e^{2(F_0+F_1+F_3)}} + \frac{1}{2}(1-a)\epsilon\right]. \tag{9}$$

The primes denote differentiation with respect to the variable x^2.

Inasmuch as the metric depends on a single variable, we can use the transformation $x^2 - \bar{x}^2$ and see to it that the condition

$$F_2 = F_0 + F_1 + F_3 \tag{10}$$

is satisfied, after which the equations become much simpler. In addition, we introduce the notation

$$\epsilon = \frac{H_0^2}{8\pi}\frac{\tilde{\epsilon}}{e^{2F_2}}$$

and measure the length in units of

$$a = \frac{c^2}{H_0\sqrt{\kappa}} .$$

As a result, Eqs. (6) - (9) take the form

$$-F''_0 = e^{2F_1} - \frac{1}{2}(1+3a)\tilde{\epsilon}, \tag{6'}$$

$$-F''_1 = -e^{2F_1} + \frac{1}{2}(1-a)\tilde{\epsilon}, \tag{7'}$$

$$-F''_3 + e^{2F_1} + \frac{1}{2}(1-a)\tilde{\epsilon}, \tag{8'}$$

$$-(F''_0 + F''_1 + F''_3) + (F'_0 + F'_1 + F'_3)^2 - (F_0'^2 + F_1'^2 + F_3'^2) = e^{2F_1} + \\ + \frac{1}{2}(1-a)\epsilon, \tag{9'}$$

where now the prime denotes differentiation with respect to the dimensionless variable $\sigma = x^2/a$. The first integrals of (6') - (8') are

$$\begin{aligned} F'_1 &= -\sqrt{n^2 - e^{2F_1} - (1-a)\int \tilde{\epsilon}\, dF_1} \\ F'_0 &= -F'_1 + 2a\int \tilde{\epsilon}\, d\sigma + n \\ F^1_3 &= -F'_1 - (1-a)\int \tilde{\epsilon}\, d\sigma + n \end{aligned} \tag{11}$$

(n - integration constant). Equation (9') yields the condition for the determination of $\tilde{\epsilon}$:

$$(1-a)\int \tilde{\epsilon}\, dF_1 + n(3a-1)\int \tilde{\epsilon}\, d\sigma - 2a(1-a)\int(\tilde{\epsilon}\, d\sigma)^{\Sigma} = a\tilde{\epsilon}. \tag{12}$$

Unfortunately, further integration is impossible in general form for an arbitrary value of α. The only exceptions are the cases $\alpha = 0$ (dustlike matter) and $\alpha = 1$.*

We consider first the case when the energy density is small (for arbitrary value of α). We then get from (12)

$$(\ \tilde{\epsilon} = \epsilon_0 \exp\left\{\frac{1-a}{a} F_1 - \frac{1-3a}{a} n\sigma\right\}, \tag{13}$$

and from (11) we get (ch ≡ cosh)

$$e^{F_1} = \frac{n}{\mathrm{ch}\, n\sigma}, \quad e^{F_2} = e^{2n\sigma}\, \mathrm{ch}\, n\sigma. \tag{14}$$

The energy density ϵ of the matter is

$$\epsilon = \tilde{\epsilon}\, \frac{H_0^2}{8\pi e^{2F_2}} = \epsilon_0 \frac{H_0^2}{8\pi} (\mathrm{ch}\, n\sigma\ e^{n\sigma})^{-\frac{1+a}{a}}, \tag{15}$$

and the density of the electromagnetic energy is

$$-T_0^0 = \frac{H_0^{\,2}}{8\pi}\, e^{2(F_1 - F_2)} = \frac{H_0^2\, n^2}{8\pi(\mathrm{ch}\, n\sigma\, e^{n\sigma})^4}. \tag{16}$$

The metric (14) can be reduced by the transformation $e^{n\sigma} \to \rho$ to Melvin's metric [1] $(x^1 \to \varphi)$**

$$-ds^2 = (\rho^2 + 1)(-dt^2 + d\rho^2 + dz^2) + \frac{\rho^2}{(\rho^2+1)^2}\, d\phi^2. \tag{17}$$

The maximum of the energy density ϵ, just as that of T_0^0, is reached on the axis when $\rho = 0$. Thus, in a self consistent magneto-gravitational field uncharged matter will be gathered and concentrated near the axis.

For the case of dustlike matter, $\alpha = 0$, Eq. (12) has only one zero solution $\tilde{\epsilon} = 0$. Thus, dustlike matter cannot be captured by a magneto-gravitational field. When $\alpha = 1$ we get from (11) and (12)

$$\tilde{\epsilon} = \epsilon_0\, e^{2n\sigma}, \tag{18}$$

$$e^{F_1} = \frac{n}{\mathrm{ch}\, n\sigma}, \quad e^{F_3} = e^{n\sigma}\, \mathrm{ch}\, n\sigma, \quad e^{F_0} = \mathrm{ch}\, n\sigma\ \exp\left(n\sigma + \frac{\epsilon_0}{2n^2} e^{2n\sigma}\right). \tag{19}$$

The energy density for the matter is in this case

$$\epsilon = \frac{\epsilon_0}{n^2} \frac{H_0^2}{8\pi} \frac{1}{(\mathrm{ch}\, n\sigma\, e^{n\sigma})^2} \exp\left(-\frac{\epsilon_0}{n^2} e^{2n\sigma}\right) = \frac{\epsilon_0}{n^2} \frac{H_0^2}{2\pi} \frac{1}{(\rho^2+1)^2} \exp\left(-\frac{\epsilon_0}{n^2}\rho^2\right) \tag{20}$$

and for the electromagnetic field

$$-T_0^\rho = \frac{H_0^2}{8\pi}(\mathrm{ch}\, n\sigma \; e^{n\sigma})^{-4}\exp(-\frac{\epsilon_0}{n^2}e^{2n\sigma}) = \frac{2H_0^2}{\pi}(\rho^2+1)^{-4}\exp(-\frac{\epsilon_0}{n^2}\rho^2) \;. \tag{21}$$

The metric for $\alpha = 1$ reduces to the form

$$ds^2 = (\rho^2+1)^2\exp(\frac{\epsilon_0}{n^2}\rho^2)(-dt^2+d\rho^2)+(\rho^2+1)^2 dz^2+\frac{\rho^2}{(\rho^2+1)^2}d\phi^2 \;. \tag{22}$$

We obtain a curious result: the presence of neutral matter with a magneto-gravitational universe decreases its characteristic dimension, in accord with (20) and (21), by a factor $\sqrt{\epsilon_0}$, where ϵ_0 is the ratio of the matter density to the electromagnetic-field density on the system axis (it is obvious that the constant n can be chosen equal to unity). Thus, the magnetic energy will be concentrated in a narrow region of small values of ρ. It is more or less evident that this result holds also for the case of any other equation of state with a parameter $\alpha \neq 1$.

This raises the question: Are quasars magneto-gravitational formations or are they only such during the initial stage of their evolution? The arguments against this assumption [1] are based on the data of Lynds and Sandayge [4], who give for the quasar M82 a characteristic field of 2×10^{-6} G in a certain effective region, whereas to explain the observed quasar dimensions fields on the order of 10^2 G are necessary. However, the presence of matter, as already shown, reduces greatly the characteristic dimension, or more accurately, the connection between the characteristic dimension and the magnetic field.

It is also possible that the magnetic field of planets is a remnant of a certain primordial magnetic field retained by the gravitational field of the nonmagnetic matter of the planets.

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* An equation of state with $\alpha = 1$ can obtain for a system of particles whose interaction is not renormalizable in quantum field theory. In this case the interaction Hamiltonian contains derivatives, and the interaction constant contains a parameter l with dimensions of length. Then, if we characterize the state of the system by a temperature T, we shall have at our disposal the dimensionless parameter $\hbar c/kTl$. Including this parameter in the dependence of the energy and of the pressure on the temperature, we can imitate the equation of state $\epsilon = p$ (see also [3]).

** It is obvious that our metric coincides in first approximation with the metric of [1], since we do not take into account the reaction of the matter on the metric.