RAMAN SCATTERING OF ULTRASOUND IN MAGNETICALLY ORDERED CRYSTALS

I. A. Akhiezer
Physico-technical Institute, Ukrainian Academy of Sciences
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We shall show that when sound oscillations propagate in magnetically ordered crystals, Raman scattering of these oscillations by spin waves is possible. The spectrum of the scattered sound can then have sharp maxima with frequencies that differ from the frequency of the incident sound by the frequency of the spin wave (magnon satellites).

We start with a system of nonlinear equations describing coupled magnetoelastic waves in ferromagnets [1]

$$\frac{d^2 u}{dt^2} - (s_l^2 - s_t^2)\,\mathrm{grad}\,\mathrm{div}\,u - s_t^2 \Delta u = \mu_i \frac{\partial}{\partial r} H_i,$$
$$\frac{d\mu}{dt} = 0, \quad \mathrm{div}\,(H + 4\pi\rho\mu) = 0, \quad \frac{\partial \rho}{\partial t} + \mathrm{div}\left(\rho \frac{du}{dt}\right) = 0, \tag{1}$$

where μ is the magnetic moment per unit mass, u the displacement vector, ρ the density of the crystal, H the magnetic field, s_l and s_t the velocities of the longitudinal and transverse sound, and $(d/dt) = (\partial/\partial t) + [(du/dt)(\partial/\partial r)]$ (we confine ourselves to the case of a small change in frequency by scattering, $\Delta\omega \ll \omega$, where ω is the frequency of the incident sound wave, and consider for simplicity a crystal with small anisotropy constant β and small magnetostriction constant f: $\beta \ll 4\pi$, $f \ll 4\pi$). Solving the equations in (1), we obtain a relation for the displacement vector u' connected with the scattered wave

$$(\omega'^2 - s_t^2 k'^2) u' - (s_l^2 - s_t^2) k'(k'u') = 4\pi\{k'k'^{-2}(qu_0)(k'M_0)(k'\delta\mu) + k k^{-2}(ku_0)(kM_0)(k\delta\mu)\}, \tag{2}$$

where M_0 is the equilibrium value of the magnetic-moment density, k' and ω' the wave vector and frequency of the scattered wave, u_0 the amplitude of the incident wave, k its wave vector, and $\delta\mu$ the fluctuation of the magnetic moment with frequency $\Delta\omega = \omega' - \omega$ and wave vector q = k' - k.

According to (2), three different Raman-scattering processes can occur in a crystal with a small magnetostriction constant: a longitudinal wave is scattered by spin waves and is transformed into a longitudinal wave ($l \to l$), a longitudinal wave is transformed into a transverse wave ($l \to t$), and a transverse wave is transformed into a longitudinal one ($t \to l$). (To investigate the fourth possible process - transformation of a transverse wave into a transverse one - we must allow in the elasticity equations for the force due to the magnetostriction; consequently the cross section of this process is proportional to $f/4\pi$.)

We shall characterize the intensity of the scattering processes by the differential scattering coefficient $d\Sigma$, which represents the increase in the energy density of the scattered wave per unit path, divided by the energy density of the incident wave. Using (2) and averaging over the fluctuations of the magnetic moment, we obtain

$$d\Sigma^{(a\to a')} = M_0^2 k'^4 s_l^{-4} \nu_{ij}^{(a\to a')} \langle \delta\mu_i \delta\mu_j \rangle_{q,\Delta\omega} \frac{d\omega' do'}{4\pi}, \tag{3}$$

where the indices a and a' denote the polarizations of the sound,

$$\nu_{ij}^{(l\to l)} = 2\kappa_i \kappa_j, \quad \nu_{ij}^{(l\to t)} = 2\sin^2\nu\cdot\cos^2\theta\cdot k^{-2} k_i k_j,$$

$$\nu_{ij}^{(t\to l)} = \sin^2\nu\cdot\cos^2\theta'\cdot k'^{-2} k_i' k_j',$$

$$\kappa = \cos\nu\cdot\cos\theta k^{-1} k - (1-\cos\nu)\cos\theta' k'^{-1} k',$$

ν is the scattering angle (angle between the vectors k and k') and $\theta(\theta')$ is the angle between the vectors k(k') and M_0 (the $\langle\ldots\rangle$ denote averaging over the fluctuations).

We now insert in (3) the well known expression for the correlator of the fluctuations of the magnetic moment per unit mass in a ferromagnet [2]

$$\langle \delta\mu_i \delta\mu_j \rangle_{q,\Delta\omega} = 2\pi h \left| \exp\frac{\hbar\Delta\omega}{T} - 1\right|^{-1} (gM_o)^2 \rho_0^{-2} \lambda_{ij} \delta(\Delta\omega^2 - \omega_q^2), \tag{4}$$

where $\omega_q = gM_0(\lambda_{11}\lambda_{22})^{1/2}$ is the frequency of the spin wave with wave vector q,

$$\lambda_{11} = \beta + \frac{H_0}{M_0} + \alpha q^2, \quad \lambda_{22} = \beta + \frac{H_0}{M_0} + \alpha q^2 + 4\pi\sin^2\chi, \quad \lambda_{12} = \lambda_{21}^* = \frac{i\Delta\omega}{gM_0}$$

(the remaining components of the tensor λ are equal to zero), g is the gyromagnetic ratio, α is the exchange-interaction constant, H_0 is the constant magnetic field, χ is the angle between the vectors q and M_0, T is the crystal temperature, and ρ_0 is the equilibrium value of its density (the axis 3 is chosen along the easy-magnetization axis, and the axis 2 is perpendicular to the (q, M_0) plane). As a result we get

$$d\Sigma^{(a\to a')} = \left| \exp\frac{\hbar\Delta\omega}{T} - 1\right|^{-1} \left(\frac{M_0}{\rho_0 s_l^2}\right)^2 \hbar g^2 k'^4 \psi^{(a\to a')} \times \delta(\Delta\omega^2 - \omega_q^2)\, d\omega' do', \tag{5}$$

where the functions ψ, whose order of magnitude is unity, are given by

$$\psi^{(l\to l)} = \kappa_i\kappa_j\lambda_{ij}, \quad \psi^{(l\to t)} = \sin^2\nu\cos^2\theta k^{-2} k_i k_j \lambda_{ij},$$

$$\psi^{(t\to l)} = \frac{1}{2}\sin^2\nu\cos^2\theta' k'^{-2} k_i' k_j' \lambda_{ij}.$$

We note in conclusion that Raman scattering of sound is characterized by a much higher relative intensity than the well known Raman scattering of electromagnetic waves by spin waves: the order of magnitude of the scattering coefficient in the case of electromagnetic waves is smaller by a factor $(cM_0)^4\rho_0^{-2}s^{-8} \sim 10^8$ than the coefficient $d\Sigma$ defined by formula (5).

[1] A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskii, JETP 35, 228 (1958), Soviet Phys JETP 8, 157 (1959).

[2] I. A. Akhiezer and Yu. L. Bolotin, Contribution the the Theory of Fluctuations and Scattering of Slow Neutrons in Ferromagnets, JETP 52, No. 5 (1967), transl. in press.

CONCERNING THE NATURE OF THE A_1 RESONANCE

N. B. Babichev, E. D. Zhizhin, and Yu. P. Nikitin
Moscow Engineering-physics Institute
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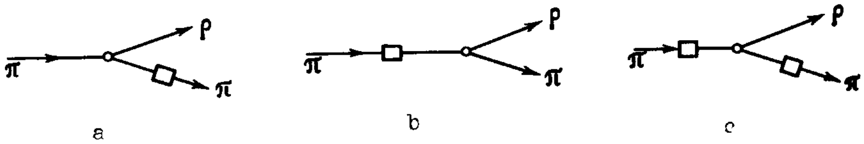
The A_1 is observed in the π-ρ system when pions interact with nucleons [1] or with nuclei [2]. The nature of this resonance has already been discussed many times in the literature. An analysis of the reaction $\pi^+ + p \to \pi^+ + \pi^- + \pi^+ + p$ on the basis of the peripheral model [3] offers evidence that the occurrence of the A_1 peak can be fully explained by scattering of a virtual pion produced as the result of the dissociation

$$\pi \to \pi + \rho \tag{1}$$

by the target proton.

We wish to call attention in this note to certain specific features of the process of diffraction dissociation (1) on nuclei, the observation of which can serve as an additional argument in favor of the diffraction mechanism for the occurrence of the A_1 peak. The dif-



ferential cross section of this process was calculated within the framework of the "black" nucleus model [4] with account of the diagrams shown in the figure. The square on the diagram corresponds to scattering of the pion by the nucleus. As shown by calculation, diagram c makes an essential contribution to the production amplitude of a ρ meson with longitudinal polarization. In spite of the fact that the wave functions of the diffracting initial and final pions differ from zero in weakly overlapping regions, the increase of the ρ-meson longitudinal polarization with increasing energy makes this diagram comparable with the corresponding contributions of diagrams a and b, and cancels them partially. In addition, in the "black" nucleus model (assuming a pointlike character of the $\rho\pi\pi$ interaction) the contributions of diagrams a and b are also partially compensated by approximately μ/m times (μ - pion mass, m - ρ-meson mass).

In the peripheral model [3], at an incident-pion energy on the order of several GeV, the main role in the dissociation (1) via the proton is played by diagram a. This is con-