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SUPERFLUIDITY OF THE COSMOLOGICAL NEUTRINO "SEA"

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In cosmological models, where states with very high density exist, the neutrino plays a very important role, if only from the point of view of formation of several chemical elements. The influence of the neutrino is particularly important in models in which a degenerate neutrino "sea" exists [1], and also apparently in the presence of considerable anisotropy during the earlier evolution stages [2].

In the isotropic model (the Friedmann model) at the earlier stages of the evolution, the total density is

$$\rho = \frac{\epsilon}{c^2} = \frac{3}{32\pi G t^2} = \frac{4{,}5\cdot 10^5}{t^2}\ \text{g/cm}^3$$

(see [3]), where t is the time reckoned from some initial instant. At the same time, the density of completely degenerate neutrinos (or antineutrinos) is

$$\rho_\nu = \frac{\epsilon_F^4}{8\pi^2 c^5 \hbar^3} \simeq 3\cdot 10^3 [\epsilon_F(\text{MeV})]^4 \text{g/cm}^3,$$

where ϵ_F is the energy at the Fermi boundary. It is obvious that $\rho_\nu \lesssim \rho$, and thus,

$$\rho_{\nu_{max}} = \frac{\epsilon_{F_{max}}^4}{8\pi^2 c^5 \hbar^3} \sim \frac{3}{32\pi G t^2},\quad \epsilon_{F_{max}} \sim \frac{3}{\sqrt{t(\text{cek})}}\ \text{MeV}. \quad (1) \tag{1}$$

We have assumed above, as usual, that the neutrinos form an ideal Fermi gas. Yet the neutrinos interact with one another, which under the conditions of a degenerate Fermi gas can lead to a superfluidity phenomenon, similar to superconductivity of electrons in a metal. The character of the interaction between the neutrinos, both electronic and muonic (we shall make no distinction between them) is still unknown. One cannot exclude the existence of a "direct" interaction between neutrinos, described by a term of the type $\lambda/2(\psi^*_\nu(\psi^*_\nu\psi_\nu)\psi_\nu)$ in the expression for the Hamiltonian. A term of similar form appears probably in the higher approximation of perturbation theory even when the neutrinos interact directly only with the electrons or the muons. In the latter case, however, we encounter patently divergent expressions, owing to the nonrenormalizability of the theory. Incidently, regardless of the origin of the term of the type $\lambda\psi^4$, no rigorous investigation of the solutions in the theory of the neutrino field, with this term taken into account, has yet been carried out (see [4,5]).

In this connection, we formulate the question of the possible superfluidity of the neutrino "sea" as follows: We use the ordinary superconductivity theory of Bardeen, Cooper, and Schrieffer in the formulation given in [6], where the interaction is described by the Hamiltonian $\lambda/2\int(\psi^*(\psi^*\psi)\psi)dz$. We disregard the appearance of Dirac matrices in this expression for the neutrinos, and assume only that the energy of the elementary excitations (neutrinos) is of the form $\xi = c(p - p_F)$, where p_F is the momentum at the Fermi boundary and c is the velocity of light (in superconductivity theory $\xi = v_F(p - p_F)$). In addition, it is easy to ascertain that, for a given momentum, the neutrino spin can have only one direction. As a result, just as in [6], we arrive at the following equation for the gap Δ in the spectrum of the system (it is assumed that $\lambda < 0$):

$$\frac{\lambda}{32\pi^3\hbar^3}\int_{-\omega}^{\Lambda}\frac{d^3p}{\sqrt{\xi^2+\Delta^2}} = -1, \quad \xi = c(p - p_F), \tag{2}$$

whereas for a superconductor

$$\frac{\lambda}{16\pi^3\hbar^3}\int_{-\omega_D}^{\omega_D}\frac{d^3p}{\sqrt{\xi^2+\Delta^2}} = -1, \quad \xi = v_F(p - p_F).$$

In (2), Λ and ω are the limits of the integration with respect to energy. The question of the choice of these limits is not yet clear. If the limits Λ and ω are large, then the neutrinos would possibly stick together (for $\lambda < 0$). At any rate, we are interested here only in the collective effect connected with the presence of the degenerate neutrino "sea." From this point of view, if there is no sticking, it hardly makes sense to use values of Λ and ω larger than the Fermi energy ϵ_F. Thus, for a tentative estimate (and we claim no more than that) we shall assume that the interaction between the neutrinos corresponds to attraction ($\lambda < 0$) and acts in an energy region such that $\Lambda = \omega = \epsilon_F$. Then we get from (2), with $\Delta \ll \epsilon_F$,

$$\Delta = 2\epsilon_F \exp\{-4\pi^2c^3\hbar^3/|\lambda|\epsilon_F^2\}. \tag{3}$$

In superconductivity theory, on the other hand, $\Delta = 2\omega_D\exp\{-(\pi^2v_F^3\hbar^3/2|\lambda|\epsilon_F^2\}$ and thus the difference in the case of the case of the neutrino consists essentially in the obvious replacement of v_F by c and, of course, in the fact that $|\lambda|$ has a different value. For superconductors with $\Delta/\omega_D \sim 10^{-2}$ and $v_F \sim 10^8$ cm/sec, we get $|\lambda| \sim 5 \times 10^{-35}$ erg-cm^3. For universal weak interaction, $|\lambda| \sim 10^{-49}$ erg-cm^3. If we assume this value in (3), then $\ln \Delta/\epsilon_F \sim -10/\epsilon_F^2$ and, for example, $\ln \Delta/\epsilon_F \sim -10$ for $\epsilon_F \sim$ erg $\sim 10^6$ MeV or $\rho \sim 3 \times 10^{27}$ g/cm^3 corresponding to $t \sim 10^{-11}$ sec (see (1)). The question of such early stages, if they are realized (this depends on the region of applicability of the employed cosmological model), is of course of great interest. If, however, the model is "hot," then the degeneracy will be incomplete or may not occur at all (see [1,3]). In addition, strictly speaking, we do not know at all the equation of state of matter when $\rho > 10^{14} - 10^{16}$ g/cm^3. On the other hand, the roughness of the calculation and the exponential character of the dependence (3), and also the uncertainty in the value of λ for the interaction in question, do not allow us to exclude likewise the possibility that the appearance of the gap is significant already at $\epsilon_F \sim 1$ MeV ($\rho \sim 3 \times 10^3$, $t \sim 10$ sec). The latter values correspond to the phase at which or near which certain nuclear reactions can occur, and the presence of a gap in the neutrino spectrum may be essential.

The superfluidity of the neutrinos could be of interest, in principle, also for sufficiently dense quasars, when their collapse or anticollapse is considered.* Such objects, however, are as yet purely hypothetical, whereas the discussion of the role of the neutrino in cosmology is quite actual. It is the latter circumstance which has induced us to call attention to the possibility of the appearance of collective (superfluid) effects for the neutrino "sea."

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* The situation is similar here to that prevailing for neutron stars [7]. In the "cold" cosmological model it would probably be necessary to consider also the superfluidity of the neutrons (for $\rho > 10^{11} - 10^{12}$ g/cm^3).