[10] S. Okubo, R. E. Marshak, and E. C. G. Sudarshan, Phys. Rev. 113, 944 (1959).
[11] H. Sugawara, Phys. Rev. Lett. 15, 870 and 977 (E) (1965).
[12] M. Suzuki, Phys. Rev. Lett. 15, 986 (1965).
[13] R. P. Feynman, Symmetries in Elementary Particle Physics, edited by A. Zichichi, pp. 161 and 477, 1965.
[14] M. A. Markov, Neitrino (Neutrinos), p. 10, 1964.
[15] I. Yu. Kobzarev and L. B. Okun', YaF 1, 160 (1965), Soviet JNP 1, 111 (1965).

THEORY OF THE DOMAIN WALL IN METALS UNDER THE CONDITIONS OF THE DE HAAS - VAN ALPHEN EFFECT

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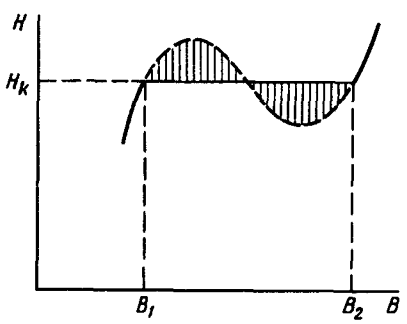
Under the conditions of the de Haas - van Alphen effect, several values of the induction B can exist on the "diagram of state" H(B) at a given value of the field H (see the figure).



The condition for the occurrence of the multiple values is the inequality $(\partial M/\partial B)_{max} > 1/4\pi$ [1]. The region of the curve between the points B_1 and B_2, determined by the equality of the shaded areas, corresponds to unstable states. It will be shown below that there can exist homogeneous phases with inductions B_1 and B_2, and that the surface tension on the interface is positive. Consequently a domain structure should appear in a sample with nonzero demagnetization factor [1].

The induction B varies in the transition region (domain wall) from a value B_1 to B_2, and therefore the connection between H and B is not the same as in the homogeneous case. The inhomogeneity must be taken into account, for if the connection between H and B were to remain constant, then the thickness of the transition layer and the surface tension would vanish. We consider essentially the case when the thickness of the transition layer is large compared with the cyclotron radius r_0, as is the case when the difference $B_2 - B_1$ is small compared with the period of the oscillations.

We calculate the magnetization increment due to the inhomogeneity in first approximation of perturbation theory, using the thermodynamic relation

$$M(x) = -\delta\Omega/\delta B(x); \quad \Omega = -T \ln Sp \exp(-H/T). \tag{1}$$

To this end, we put $B = B_0 + B'$, where B' is the inhomogeneous increment, and obtain Ω accurate to terms of second order in B'. We assume that B_0 and B' are parallel to the z axis, and that B' depends only on y.

The first-order correction to Ω is of no interest, since it does not contain the derivatives of the induction. The second-order correction is

$$\Omega^{(2)} = -\frac{e^4 B_0^2}{mc^4}\frac{L_x L_z}{(2\pi)^3}\sum_n \int dp_z \int dq \frac{1}{q^2}|B'(q)|^2\{2\omega_{n,p_z}\times$$
$$\times\left(\sum_{k=-n}^{\infty}{}' \frac{1}{k}|<n|ye^{iqy}|n+k>|^2 - \frac{1}{2m\omega_0}\right) - \frac{\partial\omega_{n,p_z}}{\partial n}|<n|ye^{iqy}|n>|^2\}. \tag{2}$$

Here $|n\rangle$ and $|n'\rangle$ are the wave functions of the oscillator centered at the point $y = 0$, $B'(q)$ is the Fourier transform of $B'(y)$, ω_{n,p_z} are the single-particle occupation numbers, and the remaining notation is standard.

Expanding the matrix elements in powers of q, we can readily verify that the total contribution from the off-diagonal and the diagonal matrix elements oscillates when B_0 is varied. The main contribution to $\Omega^{(2)}$ is made by the diagonal matrix elements; the role of the off-diagonal matrix elements reduces only to compensation of the non-oscillating term.

In the case of weak inhomogeneity we have

$$\Omega^{(2)} = -\frac{L_x L_z}{2}\frac{\partial M}{\partial B}\int dy\,[B'^2(y) - \frac{r_0^2}{4}(\frac{\partial B}{\partial y})^2 + \ldots], \tag{3}$$

from which it follows that

$$H = H_0(B) - \pi\frac{\partial M}{\partial B} r_0^2 \frac{\partial^2 B}{\partial y^2}. \tag{4}$$

The inhomogeneity energy (the part of the thermodynamic potential that depends on the derivative of the induction) can be negative (owing to the presence of the factor $\partial M/\partial B$). In this connection it is necessary to carry out an investigation of the stability against infinitesimally small inhomogeneous perturbations.

Let us consider the variation of the thermodynamic potential

$$\tilde{\Omega} = \Omega + \int dx(\frac{B^2}{8\pi} - \frac{HB}{4\pi}) = -\frac{1}{4\pi}\int dx \int B\,dH.$$

The first variation vanishes identically. Calculating the diagonal matrix elements in the quasiclassical approximation, we get

$$\tilde{\Omega}^{(2)} = \frac{L_x L_z}{2\pi}\int dq\,|B'(q)|^2\{\frac{1}{8\pi} - 2\frac{\partial M}{\partial B}\frac{1}{q^2 r_0^2}J_1^2(qr_0)\},$$

where J_1 is the Bessel function. This expression is positive if $\partial M/\partial B > 1/4\pi$, i.e., the metastability limits are the points at which $\partial M/\partial B = 1/4\pi$. This deduction can be readily generalized to the case of an arbitrary electron dispersion.

By considering a perturbation B' directed along the x axis and dependent only on J, we can obtain in analogous fashion an equation for the field H_x:

$$H_x = B_x - \frac{3\pi}{4}\bar{\chi} r_0^2 \frac{\partial^2 B_x}{\partial y^2}; \quad \bar{\chi} = \frac{M(B)}{B}. \tag{5}$$

$H_x = 0$ in the transition layer. Owing to the smallness of $\bar{\chi}$, Eq. (5) does not have in this case nonvanishing slowly varying solutions. Therefore, to calculate the domain wall it is necessary to use Eq. (4), in which the field H is set equal to H_k.

Putting for simplicity $4\pi M(B) = a \sin k\tilde{B}$, where $\tilde{B} = B - [(B_1 + B_2)/2]$ and $ak - 1 = \kappa^2 \ll 1$, we obtain the simple equation

$$-\kappa^2 \tilde{B} + \frac{k^2 \tilde{B}^3}{6} = \frac{r_0^2}{4} \frac{d^2 \tilde{B}}{dy^2}, \quad \tilde{B}(\pm\infty) = \pm \frac{\kappa\sqrt{6}}{k},$$

the solution of which is

$$\tilde{B}(y) = \frac{\kappa\sqrt{6}}{k} \,\mathrm{th}\frac{y}{2d}; \quad d = \frac{r_0}{2\sqrt{2}\kappa}. \tag{6}$$

The surface tension Δ is equal to

$$\Delta = \frac{1}{4\pi} \int_{-\infty}^{\infty} dy \left[\int_{B_1}^{B(y)} (H_0(B) - H_k)\, dB + \frac{r_0^2}{8} \left(\frac{\partial B}{\partial y}\right)^2 \right] = \frac{1}{24\pi} d\left(\frac{\partial H}{\partial B}\right)_{1,2} \times (B_2 - B_1)^2. \tag{7}$$

In the case when $B_2 - B_1$ is small compared with the period of the oscillations, including also the limiting case $(\partial M/\partial B)_{max} \gg 1$, the dimension of the domain wall is $d \sim r_0$. In this case (7) gives the correct order of magnitude of the surface energy.

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[1] J. H. Condon, Phys. Rev. 145, 526 (1965).

PERIODIC MAGNETIC STRUCTURES AND PHASE TRANSITIONS

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We shall demonstrate, using the electron diamagnetic moment as an example, a new type of phase transition (from a homogeneous structure to a periodic one), the necessary condition for which is a nonlocal connection between the thermodynamic quantities, and for which it is possible to explain the character of the singularity at the transition point. The diamagnetic moment $\vec{M}$ is determined [1,2] by the values of the magnetic induction* $\vec{B}$ over the entire Larmor orbit of radius r, and (F_m - proper free energy of the magnet)

$$\mathbf{M} = -\delta F_m / \delta \mathbf{B}, \quad \mathbf{B} = \mathbf{H} + 4\pi \mathbf{M} \tag{1}$$

are integral equations which, generally speaking, have non-growing oscillating solutions $\vec{M} = \vec{M}(\vec{r})$. (We choose the z axis in the direction of $\vec{H}$; if the electrons have an infinite mean free path, there is no dependence on z, $\vec{r} = (x, y)$, and $\vec{H}$ is a constant vector, since