$|\epsilon^1|$, corresponding to the same $|h|_{max}$ as in an isotropic crystal, is then equal to 0.4 $|\alpha|$. In spite of the absence of synchronism, ϵ^1 can reach large values, owing to the electrooptical effect. In crystals of the GaAs type $|\epsilon^1| \gtrsim 10^{-10}$ cgs esu when $\lambda_0 = 1\mu$; the length of filament formation in powerful beams ($P \sim 10^7 - 10^8$ W/cm^2) is here not larger than 1 cm.

Similar conclusions can be drawn with respect to the temporal instability of a monochromatic wave with respect to modulated perturbations (space-time self-focusing) [6].

We note that self-action is made possible by the reaction of the static field component during the detection of light. However, in view of the large values of ϵ_0 at low frequency, and in view of the absence of synchronism, the magnitude of the corresponding effects is apparently much smaller.

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* The case when the fundamental wave is extraordinary calls for a special analysis.

** It is obvious that when P is reduced it is possible to come closer to synchronism by leaving ξ small. Therefore the attained value $\epsilon^1 P \sim \epsilon^1 |A|^2$ which determines the self-focusing length is in fact proportional to $\sqrt{P}$ and not to P if ξ remains constant.

DEPENDENCE OF THE WIDTH OF A SELF-FOCUSING LIGHT BEAM ON THE POWER

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It is shown in the classical paper [1] that the geometric configuration of a self-focused cylindrical light beam is described by the normalized equation

$$d^2E^*/dr^{*2} + \frac{1}{r^*} dE^*/dr^* - E^* + E^{*3} = 0. \quad (1)$$

Equation (1) in itself does not yield an estimate of the beam width, since the normalization parameter, which is inversely proportional to the beam width ($r^* = \Gamma r$, where r is the radial coordinate) can be freely chosen from the very outset. We shall determine the beam width $\Delta \sim \Gamma^{-1}$ from the equation for the beam power

$$P = 16^{-1}\pi^{-2}n_2^{-1}\, n_0^{-1}\,\lambda^2 c\,[n_0^2 + \frac{\Gamma^2}{k_0^2}]^{1/2} A,\ k_0 = \frac{2\pi n_0}{\lambda}, \tag{2}$$

where n_0 and n_2 are the linear and nonlinear refractive indices of the nonlinear medium, λ is the wavelength of light, and the form factor A is given by

$$A = \int_0^\infty E^{*2} r^* dr^*. \tag{3}$$

We recall also that

$$\Gamma^2 = k_z^2 - k_0^2, \tag{4}$$

where k_z is the longitudinal wave number of the self-focused beam and k_0 the wave number of the plane wave.

It is seen from (2) that self focusing arises only at a beam power P exceeding the critical value $P_{cr.2}$ determined from (2) with $\Gamma = 0$, since Γ is real only when $P > P_{cr}$.

The self-focusing effect corresponds to solutions of (1) satisfying the boundary conditions

$$dE^*/dr^* = 0 \text{ if } r^* = 0,\ E^* \to 0 \text{ as } r^* \to \infty. \tag{5}$$

It is shown in [2] that Eq. (1) can have a discrete series of solutions (modes) satisfying the boundary conditions (5). This makes it possible for the energy of the focused radiation to be concentrated in the form of one or several rings around the central beam. Similar configurations were observed experimentally in [3], where reference is made also to our note [2].

The form factor of each mode must be determined by numerical integration. (For the first three modes the values obtained are $A_1 = 1.85$, $A_2 = 1.2 \times 10$, and $A_3 = 3 \times 10$.) We have accordingly a series of critical powers $P_{cr.n}$, determined in accord with (2) with $A = A_n$ and $\Gamma = 0$.

Let us trace the variation of the beam configuration with increasing beam power P. When P exceeds $P_{cr.1}$ slightly, only the first mode can exist. When $P = P_{cr.2}$ two modes can exist, one with a beam width $\Delta \sim \Gamma_{M1}^{-1}$ (Γ_{M1} is determined from (2) with $P = P_{cr.2}$ and $A = A_1$), and the other with width $\Delta \to \infty$ ($\Gamma = 0$ when $P = P_{cr.2}$ and $A = A_2$). We note that, owing to the nonlinearity of Eq. (1), the sum of two or several modes is not a solution of (1).

From considerations of the minimum energy density it can be expected that when $P = P_{cr.2}$ the first mode becomes unstable and the second mode with $\Delta \to \infty$ ($\Gamma = 0$) is realized in the experiment, i.e., in the stationary regime the beam is defocused and a plane wave $k_z = k_0$ propagates in the medium (the latter equality follows from (4) when $\Gamma = 0$). With further increase in the power P, focusing occurs again, but the beam configuration now corresponds to the second mode. At $P = P_{cr.3}$ defocusing of the second mode occurs, etc.

The concrete values of $P_{cr.n}$ and Γ depend on the parameters of the medium and are not discussed here. The values of the form factors A_n do not depend on the medium.

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IS THE EQUALITY OF THE ELECTRIC AND MAGNETIC PROTON FORM FACTORS EXACT?

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It is known that the experimentally determined proton form factors satisfy the following rule [1]:

$$G_E^p(q^2) = G_M^p(q^2) / \mu_p. \tag{1}$$

This equality is accurate to 2 - 3%. So good an agreement between the electric and magnetic form factors is quite surprising.* It is therefore of interest to assume that for some unknown reason relation (1) is exact for arbitrary q^2, and consider the experimental consequences that ensure from this assumption. We shall show below that the experimental data on $\bar{p}p$ annihilation at rest agree with this assumption.

1. If the vertex part describing the interaction between a proton and a γ quantum is written in the form $F_1(q^2)\gamma_\mu + F_2(q^2)\sigma_{\mu\nu}q_\nu/2M$ (M = proton mass), then the form factors G_E and G_M are connected with F_1 and F_2 as follows:

$$G_M = F_1 + F_2, \quad G_E = (F_1 + F_2)q^2/4M^2,$$

i.e., $G_M(4M^2) = G_E(4M^2)$. It then follows from (1) that

$$G_M(4M^2) = G_E(4M^2) = 0.$$

(Using the unitarity condition, we can show that as $q^2 \to 4M^2$ these form factors should tend to zero like $q^2 - 4M^2$ or faster.) This leads to the following behavior of the cross sections of the processes $\bar{p}p \to e^+e^-$, $\mu^+\mu^-$, and $e^+e^- \to \bar{p}p$: they are equal to zero when $q^2 = 4M^2$, increase, and then begin to drop again (at large q^2 the form factors tend to zero). If we assume that $|F_i(4M^2)| \approx |F_i(-4M^2)|$, then we can estimate the position of the maximum in these cross sections. It appears when $q \approx 5.5 - 6.0$. To observe the growth of these cross sections it is necessary to increase the accuracy by one and one-half orders of magnitude compared with that obtainable at present [1].

2. Equation (1) leads to relations between certain amplitudes of the proton annihilation process and the proton-antiproton annihilation process. Let us consider, for example, the $\bar{p}p \to 2\pi$ transition amplitude in the state $J^P = 1^-$, the form of which is $(f_1\gamma_\mu + f_2\sigma_{\mu\nu}q_\nu/2M)(k_1 - k_2)_\mu$ (where k_i are the pion momenta). The functions $f_1(q^2)$ and $f_2(q^2)$ enter