

the entire range of  $q^2$ , if only the contributions from the singularities which do not coincide in states with zero and unity isospin do not vanish.

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\* The equality (1) was obtained theoretically in a number of papers (see [2]). It should be noted, however, that it is satisfied experimentally with an accuracy higher than expected from these models.

\*\* Considerations based on the simple quark model and leading to the suppression of two-meson ionization pertain to an equal degree to the suppression of annihilation in the pair  $p\bar{p} \rightarrow \pi\rho$  [3].

#### CYCLOTRON INSTABILITY OF MONOENERGETIC IONS IN A CURVILINEAR MAGNETIC FIELD

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It is known that instabilities with characteristic frequency on the order of or higher than the ion cyclotron frequency can develop in a plasma with monoenergetic ions (see [1], and also the review [2] and the literature cited there). As a rule, these instabilities are analyzed in an approximation where a homogeneous magnetic field is assumed.

We shall show in this paper that allowance for the curvature of the magnetic field leads to one more version of cyclotron instability in a plasma with monoenergetic ions. This instability develops near the ion cyclotron frequency and its harmonics, and is remarkable in that it can exist at parameters for which other types of cyclotron instability are impossible.

To take into account the curvature of the magnetic field, let us consider the model of a helical field with cylindrical symmetry with zero shear. Assuming the oscillations to be small-scale compared with the dimension of the plasma instability, we obtain in the well-known manner (see [3]) the following dispersion equation

$$1 - \sum_{i,e} \frac{4\pi e^2}{MK^2} \int \left\{ \frac{\partial F}{\partial \epsilon} - \sum_{n=-\infty}^{\infty} \zeta_n |n|^2 G \right\} d\epsilon dv_{II} = 0, \quad (1)$$

where

$$G = \omega \frac{\partial F}{\partial \epsilon} + k_{II} \left( \frac{\partial F}{\partial v_{II}} - v_{II} \frac{\partial F}{\partial \epsilon} \right) - \frac{k_b}{\omega_B} \frac{\partial F}{\partial r},$$

$$\zeta_n = (\omega - n\omega_B - k_{II} v_{II} - k_b u_M)^{-1},$$

$$\epsilon = \frac{v_{\perp}^2}{2}, \quad \omega_B = \frac{eB}{Mc},$$

$$u_M = \frac{1}{\omega_B} \left( \frac{v_{II}^2}{R} - \frac{v_{\perp}^2}{2} \frac{\partial \ln B}{\partial r} \right),$$

$\omega$  is the perturbation frequency ( $\sim e^{-i\omega t}$ ),  $I_n = I_n(\xi)$  is a Bessel function,  $\xi = k_{\perp} v_{\perp} / \omega_B$ ,  $n$  is the number of the harmonic,  $F$  is the distribution function,  $R$  is the radius of curvature of the force line, and  $k_{\parallel}$ ,  $k_b$ , and  $k_{\perp}$  are the projections of the wave vector on the magnetic-field force line, on the binormal to it, and on the plane perpendicular to the force line. The subscripts  $i$  and  $e$  refer to ions and electrons; all other symbols are standard.

When  $R \rightarrow \infty$  and  $\partial F / \partial r = 0$  Eq. (1) goes over into the Harris equation [1].

Let us consider Eq. (1) in the case when  $k_{\parallel} = 0$ ,  $\omega \sim p\omega_{Bi}$  ( $p$  is the number of the harmonic),  $F_i = n_0(r)(1/2\pi v_{\perp})\delta(v_{\perp} - v_0)$ , and the plasma density  $n_0$  is sufficiently low so that  $\omega^2 \ll \omega_{Be}^2$  (where  $\omega_p^2 = 4\pi e^2 n_0 / M$ ). Equation (1) under precisely these assumptions, but with  $R \rightarrow \infty$ , was considered in [4,5], where it was shown that a plasma with monoenergetic ions is unstable against oscillations with  $\vec{k} \perp \vec{B}$  if the plasma density is sufficiently high. In any case, if  $\omega_{pi}^2 \leq \omega_{Bi}^2$ , then the oscillations with  $\vec{k} \perp \vec{B}$  are stable in the approximation of the homogeneous magnetic field. We shall show that allowance for the curvature of the magnetic field causes the plasma to become unstable when  $\omega_{pi}^2 \leq \omega_{Bi}^2$ . We shall assume that  $\omega \approx p\omega_{Bi}$  and retain in the sum over the harmonics only the  $p$ -th harmonic (the remaining ones are smaller by a factor  $\rho_i^2 / rR$ , where  $\rho_i = v_0 / \omega_{Bi}$  is the ion Larmor radius). We replace the frequency  $\omega$  by  $p\omega_{Bi}$  throughout, except in the denominator of the  $p$ -th harmonic.

We calculate the integral with respect to  $\epsilon$  by parts and take into account the fact that the drift velocity  $u_M$ , which enters in the denominator of the  $p$ -th harmonic, depends on  $\epsilon$ . This makes Eq. (1) quadratic with respect to the quantity  $\delta = (\omega - p\omega_{Bi} - k_b u_M) / \omega_{Bi}$ , namely

$$\delta^2 \left( \frac{\omega_{Bi}}{\omega_{pi}} \right)^2 - \delta \frac{I_p}{\xi} \left( 2p \frac{dI_p}{d\xi} + k_b \kappa \rho_i^2 \frac{I_p}{\xi} \right) - \frac{k_b p}{R} \rho_i^2 \frac{I_p^2}{\xi^2} = 0, \quad (2)$$

where  $\kappa = d \ln n_0 / dr$ , and all quantities are taken at  $v_{\perp} = v_0$ .

It is seen from (2) that when  $k_b p / R < 0$  unstable oscillations can develop, and their

maximum increment is attained when  $\xi \rightarrow \xi_p$ , where  $\xi_p$  is the point at which the quantity in the parentheses of the factor with  $\delta(I_p/\xi)$  vanishes. The order of magnitude of the increment is

$$\gamma = \text{Im } \omega \sim \omega_{pi} \rho_i \sqrt{|k_b|/R}, \quad (3)$$

with  $\text{Re } \omega = p \omega_{Bi} + (k_b/\omega_{Bi})(v_0^2/2R)$ .

We note that the cyclotron frequency which enters in the denominator of the  $p$ -th harmonic depends on the radius. Our analysis is therefore valid if its variation over one wavelength ( $\sim p_i$ ) is much smaller than the increment (3). This leads to a limitation on the plasma density at which the foregoing instability can develop:

$$(\omega_{pi}/\omega_{Bi})^2 \gg 1/|k_b|R. \quad (4)$$

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#### COALESCENCE OF BOSON REGGE TRAJECTORIES AT $t \leq 0$

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We present here a few arguments indicating that the coalescence of the Regge trajectories and their emergence to the complex plane, observed by Gribov [1] for the case of fermion trajectories, are possible also for boson Regge trajectories.

The behavior of boson Regge trajectories at  $t = 0$  ( $t$  is the square of the total energy in the annihilation channel) was considered for the case of nucleon-nucleon scattering in [2] on the basis of the requirement of analyticity of the invariant amplitudes and the assumption that the pole terms of the partial amplitudes have the structure

$$r(\sqrt{t})/j - j(t). \quad (1)$$

It was recently shown [3] that the relations obtained in [2] are the consequences of the symmetry of the  $SU(2)$  group. It has turned out here that the structure of the pole terms (1) takes place only for a particular choice of the invariants of the group. For invariants of general form, the pole terms have the following structure:

$$r(\sqrt{t})/j - j(\sqrt{t}) \pm r(-\sqrt{t})/j - j(-\sqrt{t}), \quad (2)$$