maximum increment is attained when $\xi \to \xi_p$, where ξ_p is the point at which the quantity in the parentheses of the factor with $\delta(I_p/\xi)$ vanishes. The order of magnitude of the increment is

$$\gamma = Im\,\omega \sim \omega_{pi}\rho_i\sqrt{|k_b|/R}, \tag{3}$$

with $\mathrm{Re}\,\omega = p\omega_{Bi} + (k_b/\omega_{Bi})(v_0^2/2R)$.

We note that the cyclotron frequency which enters in the denominator of the p-th harmonic depends on the radius. Our analysis is therefore valid if its variation over one wavelength ($\sim\rho_i$) is much smaller than the increment (3). This leads to a limitation on the plasma density at which the foregoing instability can develop:

$$(\omega_{pi}/\omega_{Bi})^2 \gg 1/|k_b|R. \tag{4}$$

In conclusion, I wish to thank A. B. Mikhailovskii and V. V. Arsenin for useful advice, and the members of I. N. Golovin's seminar for a discussion of the results.

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COALESCENCE OF BOSON REGGE TRAJECTORIES AT $t \le 0$

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Submitted 13 February 1967
ZhETF Pis'ma 5, No. 9, 341-344 (1 May 1967)

We present here a few arguments indicating that the coalescence of the Regge trajectories and their emergence to the complex plane, observed by Gribov [1] for the case of fermion trajectories, are possible also for boson Regge trajectories.

The behavior of boson Regge trajectories at $t = 0$ (t is the square of the total energy in the annihilation channel) was considered for the case of nucleon-nucleon scattering in [2] on the basis of the requirement of analyticity of the invariant amplitudes and the assumption that the pole terms of the partial amplitudes have the structure

$$r(\sqrt{t})\,/\,j - j(t). \tag{1}$$

It was recently shown [3] that the relations obtained in [2] are the consequences of the symmetry of the SU(2) group. It has turned out here that the structure of the pole terms (1) takes place only for a particular choice of the invariants of the group. For invariants of general form, the pole terms have the following structure:

$$r(\sqrt{t})/j - j(\sqrt{t}) \pm r(-\sqrt{t})/j - j(-\sqrt{t}), \tag{2}$$

where the ± signs depend on helicities and spins of the particles, and are determined by the symmetry of the helicity amplitudes relative to the transformation $\sqrt{t} \to -\sqrt{t}$ [4-6].

Owing to the dependence of the trajectories on $\sqrt{t}$ in (2), these trajectories coalesce at $t = 0$ and go out to the complex plane at $t < 0$.

We note that the presence of two terms in (2) does not signify doubling of the number of bound states and the resonances corresponding to the trajectories. The individual terms in (2) differ not in the set of physical states, but in the direction of propagation of the virtual particles [6].

To determine which of the modes (1) or (2) is preferable and to establish their possible interrelation, we consider a model based on the summation of the asymptotic contribution of the scattering-amplitude diagrams of particles with half-integer spin under the additional assumption that the integrals over the closed loops, which arise upon contraction of the meson lines and converge for the case of particles with zero spin, are convergent also for particles with spin. Then the diagram sum corresponding to the Regge behavior of the scattering amplitude takes the form

$$\sim s^{A(t)}, \tag{3}$$

where the matrix $A(t)$ satisfies the following conditions:

1. $A(t)$ is a relativistic invariant that depends on two sets of matrices γ_μ (for each of the fermions) and the 4-vector P_μ - the energy-momentum energy vector in the t-channel, with coefficents having no singularities at $t = 0$.

2. The matrix $A(t)$ is hermitian below the threshold of the possible reactions and at real values of the vector P_μ.

As a consequence of conditions 1 and 2, the eigenvalues of the matrix $A(t)$, corresponding to different Regge trajectories, take the form

$$\alpha_i(\pm) = a_i \pm \sqrt{b_i^2 + (c_i^2 + d_i^2)t}, \quad (i = 1,2,3,4), \tag{4}$$

where a_i, b_i, c_i, and d_i are real for values of t below threshold.

If follows from (4) that when $t \leq 0$ the Regge trajectories can coalesce pairwise, and then become complex-conjugate.

A possible picture of trajectory coalescence for the model under consideration is shown in Fig. 1, where the different trajectories are marked in accordance with mesons belonging to the trajectories, and the indices correspond to the possible types of coupling: 0 - without change of particle helicity, and ± with change of helicity by unity. For the case of equal particle masses, the trajectories P' and S' pass through the point $t = 0$ with zero residues.

As seen from Fig. 1, the coalescing trajectories have identical quantum numbers. The fact that the coalescence takes place only when $t \leq 0$ agrees with the Wigner-Neumann theorem [7]. When $t < 0$ the matrix $A(t)$ becomes non-hermitian and the conditions of the Wigner-Neumann theorem are no longer satisfied. This circumstance apparently depends little on the model used by us.

The mode (2) corresponds to the limiting case of trajectory coalescence at $t = 0$. This case corresponds to an additional symmetry which arises as a result of the vanishing of the

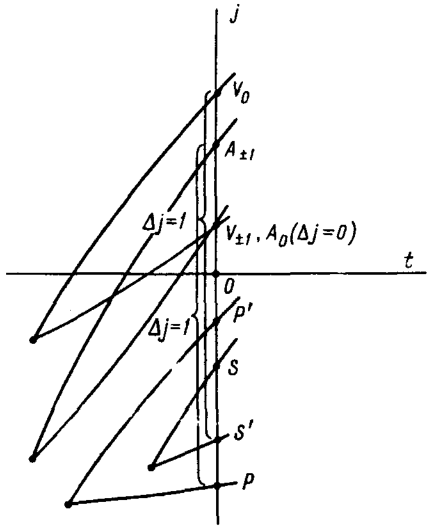

Fig. 1

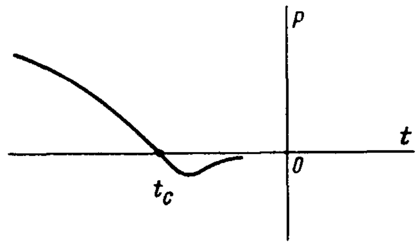

Fig. 2

b_i in (4). The degeneracy of the trajectories corresponding to this symmetry can be readily traced in Fig. 1 by letting the coalescence points approach zero.

We note in conclusion that the possibility of trajectory coalescence at $t \leq 0$, which is considered here, leads: 1) to characteristic oscillations of the differential cross section in the s-channel of the reactions [1,8], and 2) to a nonvanishing polarization that depends little on s. The qualitative dependence of the polarization on t when the trajectories coalesce at the point t_c is shown in Fig. 2. The experimental data on the polarization of the nucleons in the reaction $\pi^- + p \rightarrow \pi^0 + n$ [9] agrees with Fig. 2 if $t_c = 0$.

The author thanks V. N. Gribov for a discussion of the work.

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