FORMATION OF RESONANCES IN π^+p COLLISIONS AND REGGE POLES

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We consider here the energy dependence of the elements of polarization density matrices of the resonances produced by π^+p collisions in the reactions:

$$\pi^+ + p \to \omega^\circ + N_{33}^{*++} \quad (1) \qquad \pi^+ + p \to \rho^\circ + N_{33}^{*++} \quad (1''),$$
$$\pi^+ + p \to \pi^\circ + N_{33}^{*++} \quad (1') \qquad \pi^+ + p \to \rho^+ + p \quad (1''') \tag{1}$$

and the cross sections of these reactions at different energies, on the basis of the hypothesis of several Regge poles. The results agree well with the available experimental data at pion momenta 4 and 8 GeV/c [1-5].

The assumptions made and the calculation method are described with reaction (1) as an example. The final results are given for the remaining reactions.

According to [6], upon suitable choice of the coordinate axes, the elements of the polarization density matrix of the ω^0 meson in its rest system take the following form after summation and averaging over the spin constants of the other particles:

$$\rho^{\omega}_{\lambda\mu}(s,t) = N^{-1} \sum_{\lambda_{\bar{N}^*}\lambda_N} F_{\lambda_{\bar{N}^*}\lambda_N,\,\lambda}(s,t)\, F^*_{\lambda_{\bar{N}^*}\lambda_N,\,\mu}(s,t), \tag{2}$$

where N is a normalization factor ($\sum_\lambda \rho^{\omega}_{\lambda\lambda} = 1$) equal to $N = 32\pi s^2 d\sigma/dt$, and $F_{\lambda_{\bar{N}^*}\lambda_N,\lambda}(s, t)$ are the helicity amplitudes of the t-channel, continued into the physical region of the s-channel. Taking the amplitude factorization into account, they can be represented asymptotically, when $s \gg m^2$, as the sum of contributions of different Regge poles:

$$F_{\lambda_{\bar{N}^*}\lambda_N,\lambda}(s,t) = \sum_i \xi_i(t)\, M^i_{\lambda_{\bar{N}^*}\lambda_N}(t)\, M^i_\lambda(t) \left(\frac{s}{s_0}\right)^{\alpha_i(t)}, \tag{3}$$

where i is the number of the pole and $\xi_i(t)$ the signature factor. A similar structure is possessed by the polarization density matrix of the isobar.

In reaction (1) we took into account, besides the ρ-meson pole with $\alpha_\rho(0) \approx 0.55$ [7], also the contribution from the trajectory of the axial-vector B meson with $\alpha_B(0) \approx 0$ [8].

In the general case, the structure of the πω-meson vertex for these poles is:

$$M^\rho_\lambda(t) = a_{1\rho}(t)(\delta_{\lambda,1} + \delta_{\lambda,-1}), \quad M^B_\lambda(t) = b_{1B}(t)\delta_{\lambda,0} + c_{1B}(t)(\delta_{\lambda,1} - \delta_{\lambda,-1}) \tag{4}$$

We represent the vertex $M^\rho_{\lambda_{\bar{N}^*}\lambda_N}(t)$ in the form:

$$M^\rho_{\lambda_{\bar{N}^*}\lambda_N}(t) = a_{2\rho}(t)(\sqrt{3}\,\delta_{\lambda_{\bar{N}^*},3\lambda_N} + \delta_{\lambda_{\bar{N}^*},-\lambda_N}), \tag{5}$$

corresponding to the Stodolsky-Sakurai model [9].

We choose the vertex $M^B_{\lambda_{\bar{N}^*}\lambda_N}(t)$ in the form:

$$M^B_{\lambda_{\bar{N}^*}\lambda_N}(t) = \{b_{2B}(t)\,\delta_{\lambda_{\bar{N}^*},\lambda_N} + c_{2B}(t)(-1)^{1/2+\lambda_N}\,\delta_{\lambda_{\bar{N}^*},-\lambda_N}\}. \tag{6}$$

T a b l e

Reaction	Resonance	$\rho_{\lambda\mu}$ σ	$P_\pi = 4$ GeV/c Exptl. value [1]	$P_\pi = 4$ GeV/c Present work	$P_\pi = 8$ GeV/c Exptl. value [1]	$P_\pi = 8$ GeV/c Present work
1	N_{33}^{*++}	ρ_{33}	0.15 ± 0.04	0.15	0.24 ± 0.08	0.22
		Re ρ_{31}	-0.05 ± 0.04	≈ 0	-0.113 ± 0.083	≈ 0
		Re ρ_{3-1}	0.04 ± 0.04	0.087	0.017 ± 0.085	0.127
	ω^0	ρ_{00}	0.47 ± 0.06	0.46	0.26 ± 0.10	0.31
		ρ_{1-1}	0.13 ± 0.05	0.13	0.17 ± 0.08	0.24
		Re ρ_{10}	-0.10 ± 0.03	-0.10	-0.03 ± 0.06	-0.07
		σ	0.30 ± 0.06 [3]	0.30	0.10 ± 0.012 [2]	0.11
1'	N_{33}^{*++}	ρ_{33}	0.40 ± 0.06	0.375	0.22 ± 0.06	0.375
		Re ρ_{31}	-0.03 ± 0.07	≈ 0	0.066 ± 0.057	≈ 0
		Re ρ_{3-1}	0.21 ± 0.08	0.22	0.132 ± 0.067	0.22
		σ	0.29 [4]	0.21	0.11 ± 0.01 [2]	0.11
		$\sigma(3.5)$	0.20 ± 0.04 [11]	0.23	-	-
1''	N_{33}^{*++}	ρ_{33}	0.08 ± 0.03	0.08	0.05 ± 0.03	0.05
		Re ρ_{3-1}	0.01 ± 0.03	≈ 0	0.015 ± 0.028	≈ 0
		Re ρ_{31}	-0.01 ± 0.03	-0.01	-0.076 ± 0.033	-0.01
	ρ^0	ρ_{00}	0.77 ± 0.04	0.78	0.77 ± 0.04	0.84
		ρ_{1-1}	-0.04 ± 0.03	-0.04	-0.035 ± 0.024	-0.01
		Re ρ_{10}	-0.06 ± 0.03	-0.06	-0.119 ± 0.025	-0.05
		σ	0.60 ± 0.18 [5]	0.95	0.43 ± 0.04 [2]	0.40
1'''	ρ^+	ρ_{00}	0.70 ± 0.08	0.70	0.54 ± 0.07	a) 0.54 b) 0.56
		ρ_{1-1}	0.17 ± 0.08	a) 0.15 b) 0.10	0.075 ± 0.062	a) 0.23 b) 0.18
		Re ρ_{10}	-0.07 ± 0.07	-0.07	-0.08 ± 0.05	a) -0.05 b) -0.06
		σ	0.34 [4]	0.34	0.12 ± 0.04 [2]	a) 0.11 b) 0.105

From expressions (2) - (6) we find that the elements of the ω^0-meson and isobar polarization density matrices averaged over t can be written in the form:

$$\bar{\rho}^{\omega}_{00} = \frac{2}{\bar{N}}\left(\frac{s}{s_0}\right)^{2\alpha_B(0)} g_0 ; \quad \bar{\rho}^{\omega}_{11} = \frac{8}{\bar{N}}\left(\frac{s}{s_0}\right)^{2\alpha_\rho(0)} g_1 + \frac{2}{\bar{N}}\left(\frac{s}{s_0}\right)^{2\alpha_B(0)} g_2 ;$$

$$\bar{\rho}^{\omega}_{1-1} = \frac{8}{\bar{N}}\left(\frac{s}{s_0}\right)^{2\alpha_\rho(0)} g_1 - \frac{2}{\bar{N}}\left(\frac{s}{s_0}\right)^{2\alpha_B(0)} g_2 ; \quad \bar{\rho}^{N^*}_{33} = \frac{6}{\bar{N}}\left(\frac{s}{s_0}\right)^{2\alpha_\rho(0)} g_1 ; \tag{7}$$

$$\bar{\rho}^{N^*}_{11} = \frac{2}{\bar{N}}\left(\frac{s}{s_0}\right)^{2\alpha_\rho(0)} g_1 + \frac{2}{\bar{N}}\left(\frac{s}{s_0}\right)^{2\alpha_B(0)} g_2 + \frac{1}{\bar{N}}\left(\frac{s}{s_0}\right)^{2\alpha_B(0)} g_0 ; \quad \bar{\rho}^{N^*}_{31} \approx 0; \quad \bar{\rho}^{N^*}_{3-1} = \frac{2\sqrt{3}}{\bar{N}}\left(\frac{s}{s_0}\right)^{2\alpha_\rho(0)} g_1 ,$$

where $\bar{N} = 32\pi s^2\sigma(s)$ and the quantities g_0, g_1, and g_2 are given by integrals of the type

$$g_1 = \int_{t_0}^{-|t|_{min}} |\xi_\rho(t)|^2 a_{1\rho}^2(t)\, a_{2\rho}^2(t) \exp\{2a_\rho^1(o)\, t \ln(\frac{s}{s_0})\}\, dt, \tag{8}$$

the g_i being functions of $\ln(s/s_0)$. We assume that when p_π goes from 4 to 8 GeV/c we can neglect the variation of g_i, i.e., a more important role is played by the power-law dependence on s in the elements of the polarization density matrices. Under this assumption we obtain the additional relation $\bar{\rho}_{10}^{\omega}(s)/\bar{\rho}_{10}^{\omega}(s_0) = \bar{\rho}_{00}^{\omega}(s)/\bar{\rho}_{00}^{\omega}(s_0)$. By determining the g_i from the experimental data at $p_\pi = 4$ GeV/c we can obtain $\rho_{\lambda\mu}^{i}$ and the cross section of the reaction at $p_\pi = 8$ GeV/c. The results of the calculation are listed in the table (the underscored quantities were used to determine the parameters).

In reaction (1'') we took into account the contribution of the ρ-meson trajectory. A similar analysis was made in [10]. It is important to note that within the framework of the given model the elements $\rho_{\lambda\mu}^{N^*}$ should not depend on s. The values of $\rho_{\lambda\mu}^{N^*}$ listed in the table for reaction (1') agree well with the experimental data at a pion momentum $p_\pi = 3.54$ GeV/c [11].

The experimental data on the reaction (1'') can be satisfactorily described by taking into account the pion and A_2-meson Regge poles. The contribution of the pion is ~90% at $p_\pi = 4$ GeV/c. The slow decrease of the cross section with energy is connected with the appreciable reduction in the value of $|t|_{min}$ when p_π goes from 4 to 8 GeV/c.

Contributing to the reaction (1''') are the π-, A_2-, φ-, and ω-meson poles. In the calculations we put for simplicity $\alpha_{A_2}(0) = \alpha_\varphi(0) = \alpha_\omega(0) = 0.5$ and two solutions are presented, corresponding to different choices of the element $\bar{\rho}_{1-1}$ at a pion momentum $p_\pi = 4$ GeV/c.

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