

$\Delta\nu \approx \Delta\nu_0\sqrt{\alpha}$  ( $\Delta\nu_0 \approx 0.5 \text{ cm}^{-1}$  is the active-medium luminescence line width, and  $\alpha \approx 0.5 \text{ cm}^{-1}$  is the gain per unit length at threshold).

5. It is seen from the foregoing that radiation-intensity fluctuations, similar to the fluctuations of black-body radiation, occur in a laser with nonresonant feedback. This makes it possible to set up experiments on the correlation of photons (similar to those of Brown and Twiss [8]).

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#### CONTRIBUTION TO THE THEORY OF HOLOGRAPHIC IMAGE MAGNIFICATION

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Image magnification is attained in holography by using diverging beams. This makes a holographic microscope feasible. Usually [1] one uses for this purpose the hologram of the object itself. We call attention to another type of microscope, in which the hologram records not the object, but interference of waves from two sources. By viewing through this hologram the object illuminated by a coherent beam, it is possible to obtain a magnified image of the object. This differs in essence little from the use of a Fresnel zone plate as a lens. Such a scheme may turn out to be convenient under certain conditions.

Let us sketch briefly the theory of such a microscope. If A and B are coherent sources, then the field at the point P of a photographic plate at  $z = 0$  will be  $U(P) = U_A + U_B = \exp(ik \cdot PA) + \exp(-ik \cdot PB)$ . Assume that a coherent light field  $s(O)$  with wave number  $k'$  is incident from the object O onto a developed hologram plate magnified  $m$  times. Then the field in a certain plane I will be

$$S(I) = \int s(O) e^{-ik \cdot OP} |U(m^{-1}P)|^2 e^{-ik' \cdot PI} dO dP. \quad (1)$$

We confine ourselves henceforth only to the field  $S(I)$  due to the interference terms  $U_A^* U_B$  or  $U_A U_B^*$ , assuming that the fields from  $|U_A|^2 + |U_B|^2$  will turn out to be in other parts of the plane I. Then, in the narrow-beam approximation, assuming for simplicity that the object is

one-dimensional, we have

$$S(x_I) = \int s(x_0) \exp \left\{ -ik' \left[ \frac{(x_P - x_0)^2}{2z_0} + \frac{(x_I - x_P)^2}{2z_I} \right] \pm \right. \\ \left. \pm ik \left[ \frac{(m^{-1}x_P - x_A)^2}{2z_A} - \frac{(m^{-1}x_P - x_B)^2}{2z_B} \right] \right\} dx_0 dx_P. \quad (2)$$

We stipulate that the coefficients of  $x_P^2$  in the exponential vanish, and then the integral with respect to  $dx_P$  goes over into a  $\delta$ -function of  $x_0$ , and after integrating with respect to  $x_0$  we

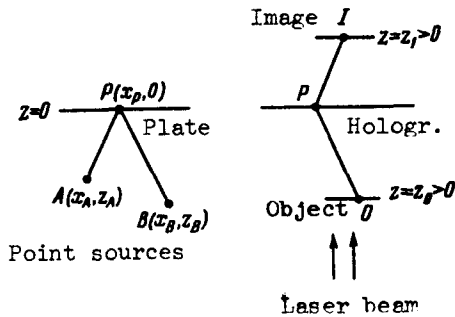


Fig. 1

find that the field  $S$  duplicates the field  $s$  to a certain scale, i.e., the image duplicates the object. The argument of the  $\delta$ -function is the coefficient of  $x_P$  (2), so that the conditions on the plane  $I$  are as follows:

$$-k' (z_0^{-1} + z_I^{-1}) \pm km^{-2} (z_A^{-1} - z_B^{-1}) = 0,$$

$$k' \left( \frac{x_0}{z_0} + \frac{x_I}{z_I} \right) \mp \frac{k}{m} \left( \frac{x_A}{z_A} - \frac{x_B}{z_B} \right) = 0.$$

From this we get for the position of the image  $I$  and for the magnification  $M$ :

$$z_I = \left[ -\frac{1}{z_0} \pm \frac{k}{k'm^2} \left( \frac{1}{z_A} - \frac{1}{z_B} \right) \right]^{-1}, \quad M = \left[ 1 \mp \frac{kz_0}{k'm^2} \left( \frac{1}{z_A} - \frac{1}{z_B} \right) \right]^{-1}.$$

The field in the plane  $I$  will be

$$S(x_I) = s \left( -\frac{x_I}{M} \mp \frac{kz_0}{mk'} \left( \frac{x_B}{z_B} - \frac{x_A}{z_A} \right) \right)$$

with a certain phase factor, which we do not observe visually. There are always two images; sometimes both are virtual, sometimes one is virtual and the other real, and sometimes both are real. Being in different planes, they do not interfere with each other.

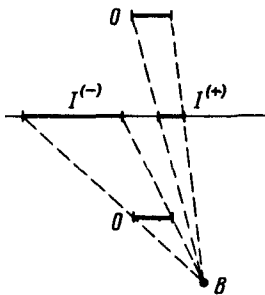


Fig. 2

In the simplest case when  $m = 1$ ,  $k = k'$ , and  $z_A = \infty$  we have

$$M = \frac{z_B}{z_B \pm z_0}; \quad z_I = -\frac{z_0 z_B}{z_B \pm z_0}; \quad x_I = \frac{x_0 z_B \pm z_0 x_B}{z_B \pm z_0}.$$

A magnified image is obtained when the object is placed only slightly closer to the hologram than the source  $B$ . Figure 2 shows how the size of the image can be determined graphically.

It is thus clear that to observe an object in a holographic microscope there is no need for preparing a hologram of the object; it suffices to prepare a hologram of two point sources and this hologram is itself the microscope.

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