If we put

$$2\sigma_{-1}(I=1/2) - \sigma_{-1}(I=3/2) = 2\sigma_{-1}(\gamma, p) + [2w(1/2) - w(3/2)]\sigma_{-1}(\gamma, n), \qquad (4)$$

where σ_{-1} are integrals similar to (1) of the corresponding cross sections and w(I) is the averaged probability of encountering the value of the isospin I in channel (3), then, assuming w(1/2)/w(3/2) = 0, 0.1, 0.3, and 0.5, we get in the right side of (1) the values 11.6, 14.2, 18.3, and 21.4 mb. The value of the left side is in this case 13.7 mb. Thus, assumptions of quite general character lead to a result of practical significance, namely that the connection between channels (2) and (3) via the unitarity condition is immaterial. A study of the mechanism of intensification of the transitions in the state with I = 3/2 in the three-pacticle reaction channel (3), predicted on the basis of the sum rule, may be of interest in connection with the problem of the "trineutron" n^3, which has been widely discussed in the recent literature (see [7]), and the problem of excited levels in three-body nuclei.

In conclusion, I am grateful to A. M. Baldin and V. N. Fetisov for interest in this work.

[1] N. Cabibbo and L. Radicati, Phys. Lett. 19, 697 (1966).
[2] V.N. Gribov, B. L. Ioffe, and V. M. Schekhter, Phys. Lett. 21, 457 (1966).
[3] R. N. Faustov, R. E. Kallosh, and V. G. Pisarenko, JINR Preprint E-2865, Dubna, 1966.
[4] F. J. Gilman and H. J. Schnitzer, Phys. Rev. 150, 1362 (1966).
[5] A. T. Varfolomeev and A. N. Gorbunov, JETP 47, 30 (1964), Soviet Phys. JETP 20, 20 (1965).
[6] H. Collard, R. Hofstadter, E. B. Hughes, A. Johansson, M. Yearian, R. B. Day, and R. T. Wagner, Phys. Rev. 138, B57 (1965).
[7] K. Okamoto and B. Davies, Phys. Lett. 24B, 18 (1967).

NONLINEAR QUANTUM PSEUDORESONANCE IN METALS

M. Ya. Azbel' and L. B. Dubovskii
Institute of Theoretical Physics, USSR Academy of Sciences
Submitted 21 March 1967
ZhETF Pis'ma 5, No. 11, 414-417 (1 June 1967)

Nonlinear effects in metals placed in an electromagnetic field are usually small. In a quantizing constant magnetic field H, however, as is well known (see [1]), the magnetic moment M and the conductivity σ oscillate with a frequency $cS/e\hbar H \gg 1$. As noted in [2], a nonlinearity appears in these oscillating terms even in a relatively weak alternating field. To this end it is sufficient to have an external-field amplitude H_1 on the order of the period of the quantum oscillations, i.e., $H_1 \gtrsim H(e\hbar H/cS)$(S - area of the Fermi-surface section).

To observe quantum oscillations it is necessary to have a sufficiently strong magnetic field ($\hbar\Omega > 2\pi^2 kT$, $\Omega\tau > 1$, the latter is equivalent to $r < \ell$. Here $\Omega = eH/m^*c$, τ and ℓ are the time and mean free path of the electrons, and r their Larmor radius).

Nonlinearity takes place at all frequencies. The effect reaches a maximum at $\omega \ll 1$ (ω is the frequency of the alternating field), when the system is able to "attune itself" to the alternating field existing at the given instant, and can "follow" the frequency.

For simplicity of calculation we assume that the normal skin effect conditions are satisfied: $\delta \gg \ell$, and therefore also $\delta \gg r$ (δ - thickness of skin layer). At distances r and ℓ we can therefore assume the field to be homogeneous and all the relations to be local. It is therefore possible to use in the main approximation the formulas obtained for the static case.

We assume that the magnetic susceptibility is small* (i.e., according to [2], that $(V_F/c)^2(cS/e\hbar H)^{3/2} << 1$), so that the magnetic moment can be regarded in Maxwell's equations as a perturbation. Then, in the main approximation, the alternating electromagnetic field in the metal has the usual form

$$E = E_0 e^{-z/\delta}\cos(\omega t - z/\delta), \quad H_1 = H_{10} e^{-z/\delta}\cos(\omega t - z/\delta),$$

and in the next approximation we have in Maxwell's equations the magnetic field in the specified zeroth-approximation field, i.e.(since, as stated above, we can use the static formulas), we get $M = M_0\cos(\kappa H_1 + \beta)$ when $(cS/e\hbar H)(H_1/H)^2 << 1$, where $\beta = cS/e\hbar H$, $\kappa = cS/e\hbar H \sin\alpha$, and α is the angle between the direction of H and the normal to the metal surface.

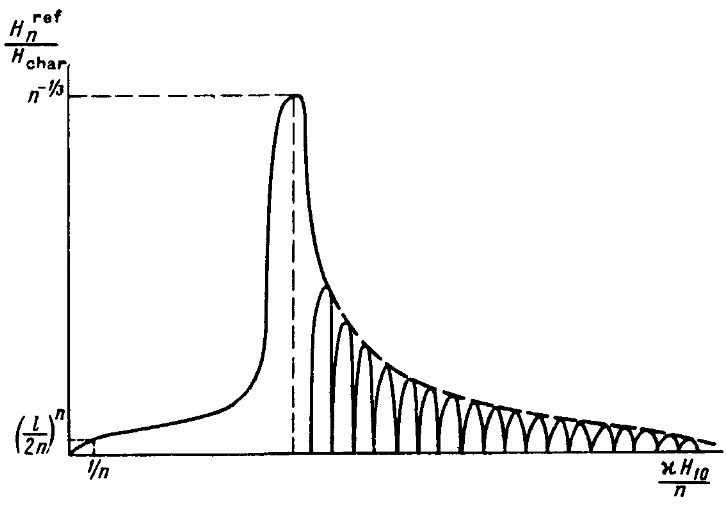
In order for the nonlinear effects to be appreciable, we obviously must have $\kappa H_{10} >> 1$. In this case the value of M oscillates rapidly with depth at distances $\delta/\kappa H_{10}$ which are small compared with the depth δ of the skin layer. Therefore, naturally, the field intensities on the surface will be dtermined only by the value of the magnetic moment on the surface: $M = M_0\cos(\kappa H_{10}\cos\omega t + \beta)$.

In particular, the reflection coefficient of the n-th harmonic is connected with the n-th coefficient of the expansion of the function $\cos(\kappa H_{10}\cos\omega t)$, i.e., with

$$c_n = 1/\pi \int_0^{2\pi} \exp[i(\kappa H_{10}\cos x - nx)\,dx = \exp[-(\pi n/2)i]J_n(\kappa H_{10}).$$

It is easy to see that several cases are possible when $\kappa H_{10} >> 1$. If $n > \kappa H_{10}$, then c_n decreases exponentially with change of $n/\kappa H_{10}$; if $n < \kappa H_{10}$, then $c_n \sim 1/\sqrt{n}$. Finally, when $n = \kappa H_{10}$ we have a peculiar "pseudoresonance": $c_n \sim n^{-1/3}$, with a relative "half-width" on the order of $n^{-2/3}$. As is clear from the foregoing, this "pseudoresonance" has essentially a quantum nonlinear character.

Let us estimate the amplitude of the resonant harmonic. First, it is proportional to the amplitude of the magnetic moment, the order of which (see [1]) is $M \sim H(V_F/c)^2(cS/e\hbar H)^{1/2}$. Second, a factor $\sqrt{\omega/\sigma}$ appears as a result of the nonlinearity of the effect under consideration (we get from Maxwell's equations that the electric component of the field in the metal is $E \sim \sqrt{\omega/\sigma}M$; at the n-th harmonic there is no incident wave, only a reflected one, and therefore



the electric and magnetic field components are equal at the boundary; this gives rise to the factor $\sqrt{\omega/\sigma}$ for the magnetic field component). It is important to note that the conductivity σ, which enters in the equations, has a different order of magnitude than the conductivity σ_0 in the absence of a magnetic field (see [3]). If the sections of the Fermi surface are closed curves, then, as shown by calculation, $\sigma \sim (r/\ell)\sigma_0$ if the number of electrons is not equal to the number of holes, and $\sigma \sim (r/\ell)^2\sigma_0$ if the number of electrons is equal to the number of holes, if H is not parallel to the surface of the metal. The width of the resonance peak is $n^{-2/3}$. When $\kappa H_{10}/n < 1$ there are no oscillations and the curve falls off exponentially. When $\kappa H_{10}/n > 1$ there are rapid oscillations. We present the calculated amplitude of the n-th reflected wave, H_n^{ref}, in different regions:

$$H_n^{ref} = H_{char}\begin{cases} \dfrac{1}{n!}\,\dfrac{1}{n+\sqrt{n}}\left(\dfrac{\kappa H_{10}}{2}\right)^n & (\kappa H_{10} \ll 1) \\ \left(\dfrac{\kappa H_{10}}{2n}\, e\right)^n e^{-(\kappa H_{10})^2/2n} & (1 \ll \kappa H_{10} \ll n) \\ \sqrt{\dfrac{2}{\pi\kappa H_{10}}}\,(-\xi)^{-1/4}\, e^{-n(-\xi)^{3/2}/3} & (n^{-2/3} \ll -\xi \ll 1) \\ \dfrac{\Gamma(1/3)}{2\pi\sqrt{3}}\,\sqrt[3]{\dfrac{6}{n}} & (|\xi| \ll n^{-2/3}) \\ \sqrt{\dfrac{2}{\pi\kappa H_{10}}}\,\xi^{-1/4}\cos\left(\dfrac{1}{3}n\xi^{3/2} - \dfrac{\pi}{4}\right) & (n^{-2/3} \ll \xi \ll 1) \\ \dfrac{n}{(\kappa H_{10})^{3/2}}\sqrt{\dfrac{2}{\pi}}\cos\left(n\kappa H_{10} - \dfrac{n\pi}{2} - \dfrac{\pi}{4}\right) & (n \ll \kappa H_{10}) \end{cases}$$

$$\xi = 2\left(1 - \frac{\kappa H_{10}}{n}\right)$$

$$H_{char} = H_1^{ref}\Big|_{\kappa H_{10}=1} \sim \sqrt{\frac{\omega}{\sigma}}\left(\frac{v_F}{c}\right)^2\left(\frac{cS}{e\hbar H}\right)^{1/2} H.$$

The derivation of the formulas and a discussion of the results will be the subject of a separate article.

[1] I. M. Lifshitz and M. I. Kaganov, UFN 78, 411 (1962), Soviet Phys. Uspekhi 5,878 (1963).
[2] M. Ya. Azbel' and G. A. Begiashvili, JETP Letters 3, 201 (1966), transl. p.128.
[3] I. M. Lifshitz and M. I. Kaganov, UFN 87, 389 (1965), Soviet Phys. Uspekhi 8, 805 (1966).

*We refer to the differential quantum magnetic susceptibility [2].

ERRATA

Volume 5, No. 11

In the article by A. A. Galkin et al. (p. 324), the upper figure should be labeled "a" and the lower "b."

In the article by M. Ya. Azbel' et al. (p. 340, 10th line from top), replace "... in different regions:" by "...in different regions (see the figure):"