[1] H. Welker, Z. Naturforsch. 6a, 184 (1951); O. Madelung, L. Tewordt, and H. Welker, ibid. 10a, 476 (1955).
[2] K. Ando, J. Phys. Soc. Japan 21, 1295 (1966).

QUANTUM SIZE EFFECTS IN THE ELECTRIC CONDUCTIVITY OF THIN FILMS

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We report here the calculation of two quantum size effects in the kinetic characteristics of thin films: 1) oscillations of the resistivity ρ with the film thickness L; 2) oscillatory dependence of the current density j on the electric field intensity E (quantum corrections to Ohm's law). We note that size effects in Bi films were observed recently [1,2]; the theory of these effects was considered in [3,4]. Sandomirskii's paper [4] contains some results pertaining to the first of the aforementioned size effects (oscillations of ρ with L). Unlike his work, in which the calculation is based on the use of the kinetic equation, we shall use quantum field-theoretical methods [5], which yield the connection between j and E for small values of these quantities. Specular reflection of the electrons from the film boundaries is assumed, the resistance being due to scattering by point impurities with a volume mean free path greatly exceeding the film thickness.

The Hamiltonian of the system is of the form $H = H_0 + H_1$, where

$$H_0 = \sum_\alpha \epsilon_\alpha a_\alpha^+ a_\alpha, \quad H_1 = g \sum_{\alpha \neq \alpha'} a_{\alpha'}^+ a_\alpha . \tag{1}$$

Here $\alpha = (p_x, p_y, n)$ is the aggregate of the quantum numbers of the electron in the film, $\epsilon_\alpha = (\vec{p}^2/2m) + (\pi^2 n^2/2mL^2)$ (it is assumed also that H_0 contains implicitly the interaction between the electrons). H_1 is the effective electron-impurity interaction Hamiltonian [6]. According to [6], the interaction constant g is connected with the free-path time $\tau = \tau_{tr}$ by the relation $g = (\pi\nu/mp_0)^{1/2}$, $\nu = 1/\tau$.

In the zeroth approximation ($H_1 = 0$) we consider a state with specified current $j = Nev_T$ (shifted Fermi surface), where v_T is the transport velocity and N is the electron density. There is no electric field in this system. When H_1 is included, dissipation of the momentum takes place, and therefore conservation of the stationary state calls for creation of an electric field $\vec{E}$, defined by

$$NeE = -\left\langle \frac{dP}{dt} \right\rangle = -i\langle [H_1 \times P] \rangle, \quad P = \sum_\alpha p a_\alpha^+ a_\alpha \tag{2}$$

($\vec{E}$, $\vec{j}$, and $\vec{p}$ lie in the plane of the film).

Commuting H_1 with $\vec{P}$ and calculating the resultant mean values $\langle a_{\alpha'}^+ a_\alpha \rangle$ in the first nonvanishing approximation in H_1,, we arrive at the formula

$$NeE = 2\pi g^2 \sum_{\alpha\alpha'} (p - p')[f_0(\epsilon_\alpha - pv_T) - f_0(\epsilon_{\alpha'} - p'v_T)]\delta(\epsilon_\alpha - \epsilon_{\alpha'}), \tag{3}$$

where $f_0(\varepsilon) = [\exp(\varepsilon - \mu/T) + 1]^{-1}$ is the Fermi distribution function.

It is easy to verify that when summation over n and n' is replaced in the resultant expression by integration, we get the usual Ohm's law $\vec{E} = \rho_0 \vec{j}$, where $\rho_0 = m\nu/Ne^2$. The quantity

ρ_0 depends here on L, meaning that there is no classical size effect (if the dispersion is quadratic and the reflection is specular) [7]. The differences between the sums and the integrals necessitate, as usual, quantum corrections that can be easily calculated with the aid of the Poisson formula [8]. If we consider the temperature region $T \gtrsim \delta E$, where $\delta E = \pi p_0/mL$ is the distance between the quantized levels on the Fermi surface, and putting $\delta E \ll \mu$, we can confine ourselves to the first oscillation harmonic (s = 1). In this approximation the resistivity $\rho(j) = E/j$ takes the form

$$\rho = \rho_0 \{1 + \frac{2\pi T}{\mu} \exp[-\frac{\pi p_0 L T}{\mu}] \sin 2p_0 L \phi(\frac{2mLj}{Ne})\}, \tag{4}$$

where p_0 is the Fermi momentum and $\phi(t)$ is given by

$$\phi(t) = \frac{3}{t} \int_0^{\pi/2} J_1(t \sin\phi) \sin^2\phi \, d\phi = \frac{3}{t^3}(\sin t - t\cos t) \tag{5}$$

The limit imposed on the excitation lifetime by volume and surface scattering gives rise to an additional factor [3] $p_0\exp(-2mL/p\tau)$ in front of the oscillating term in (4), with p the phenomenological specularity parameter ($p \leq 1$).

Since $\phi = 1$ when $t = 0$, we get a linear connection between E and j at low currents. We see from (4) that in this case the resistance oscillates with L at a period π/p_0, their relative amplitude being determined by the factor $\delta E/\mu \sim 1/p_0 L$ (if $T \sim \delta E$). With increase in j, the connection between E and j becomes nonlinear and the resistance begins to oscillate with change of j (or E)*. The current at which such deviations from Ohm's law become appreciable is given by the condition

$$Lj \gtrsim Ne\hbar/m \tag{6}$$

(as follows from (4), the oscillations can have a noticeable amplitude here if L is of the same order as the de Broglie wavelength p_0^{-1}). Putting $N \sim 10^{-6}$ el/atom and $m \sim 10^{-2} m_0$ (tentatively for Bi) we get for the required values of Lj

$$Lj \sim 1 - 10 \text{ A/cm}.$$

Physically condition (6) means that the energy acquired by the electron through the drift ($p_0 v_T$) becomes equal to the distance δE between quantum levels (in our quasiclassical approximation this is much lower than the Fermi energy μ). Study of the oscillating dependence of the resistance on the current makes it possible to observe quantum size effects in one sample, without changing its thickness L. We note also that a similar possibility is afforded by a study of the pressure dependence of the oscillations due to the size effect.

[1] V. N. Lutskii, D. N. Korneev, and M. I. Elinson, JETP Letters 4, 267 (1966) transl.p.179.
[2] Yu. F. Orgin, V. N. Lutskii, and M. I. Elinson, JETP Letters 3, 114 (1966), transl.p.71.
[3] G. A. Gogadze and I. O. Kulik, FTT 7, 432 (1965), Soviet Phys. Solid State 7, 345(1965).
[4] V. B. Sandomirskii, JETP 52, 158 (1967), Soviet Phys. JETP 25, 101 (1967).
[5] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskii, Metody kvantovoi teorii polya v statisticheskoi fizike (Quantum Field Theory Methods in Statistical Physics), 1962.
[6] I. O. Kulik, K teorii rasprostraneniya ul'trazvuka v metallakh (On the Theory of Ultrasound propagation in metals), VINITI, 1965.
[7] A. N. Friedman and S. H. Koenig, IBM Jour. 4, 158 (1960).
[8] I. M. Lifshitz and A. M. Kosevich, JETP 29, 730 (1955), Sov. Phys. JETP 2,636 (1956).

*We note that this effect is similar to the oscillation of the tunnel current from films with variation of the voltage V, discussed by us earlier [3].