strange. The parity is $P = (-1)^L$ with two doubtful exceptions. It is probable that $|k| = L$. Classically, the rotation energy $E = L^2/2I$ is minimal when the moment of inertia I is maximal. Rotation around the axis with small I is unstable under perturbations. For the triangle, the maximum I is about the ζ axis perpendicular to the plane. Only rotation around ζ is stable. From the quantum point of view this means $|k| = L$. As is well known, the average energy of quantized rotation is

$$\langle H \rangle = (q+2)/4\,[L(L+1) - k^2] + s k^2/2 \tag{4}$$

q, r, and s are inversely proportional to the moments of inertia. s is about the ζ axis. The energy is minimal when $|k| = L$. Classically, the quarks rotate in one plane. The z axis is parallel to ζ.

[1] Ya.I.Azimov, V.V.Anisovich, A.A.Ansel'm et al. JETP Letters 2, 109, 1965, transl. p. 68.

EXCITATION OF OSCILLATIONS IN "DECAY" INSTABILITY OF STEADY-STATE WAVES IN A PLASMA

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This paper is devoted to an investigation of "decay" instability [1,3] of waves in nonlinear media. For concreteness we consider waves in a plasma, although instabilities of this type can occur also for waves in other nonlinear media - in solids, for capillary waves on the surface of a liquid, etc.

We shall show that decay instability can give rise to "violet" satellites that grow with increments proportional to the amplitude of the initial wave. In addition, it will be made clear that under definite conditions the "decay" instability does not arise even upon satisfaction of the decay conditions*

$$\omega_k = \omega_{k'} + \omega_{k''}, \qquad \mathbf{k} = \mathbf{k}' + \mathbf{k}''. \tag{1}$$

To demonstrate this, we consider a simple example in which we have, besides the waves $\vec{k}'$ and $\vec{k}''$, another wave $\vec{k}'''$ whose frequency and wave vector satisfy the relations

$$\omega_{k'''} = \omega_k + \omega_{k''}, \qquad \mathbf{k}''' = \mathbf{k} + \mathbf{k}''. \tag{2}$$

In addition, we assume that the phases of all the waves are correlated.

As is well known [2], in first approximation of perturbation theory (with respect to the wave amplitudes) the time evolution of the wave amplitudes C_ℓ is described by the equations

$$\frac{\partial C_{k'}}{\partial t} = -iV_{k'kk''_-}\, C_k C_{k''_-},$$

$$\frac{\partial C_{k''_-}}{\partial t} = -iV_{k''_-k_-k'}\, C_{k_-} C_{k'} - iV_{k''_-kk'''_-} C_k C_{k'''_-}, \tag{3}$$

$$\frac{\partial C_{k'''_-}}{\partial t} = -iV_{k'''_-k_-k''_-}\, C_{k_-} C_{k''_-},$$

where the concrete form of the matrix elements $V_{\ell mn}$ describing the interaction of the waves under consideration is of no importance in what follows. Neglecting the time dependence of $C_k^{(0)}$, we arrive at a system of linear differential equations with constant coefficients, so that the solution can be sought in the form $e^{\nu t}$. The characteristic equation for the determination of ν has in this case the form

$$\nu^2 = -|C_k^{(0)}|^2 V_{k'kk''_-} V_{k''_-k_-k'} - |C_k^{(0)}|^2 V_{k'''_-k_-k''_-} V_{k''_-kk'''_-}. \tag{4}$$

We see that the resultant correction to the characteristic frequency is

$$\nu^2 = \nu_{1,2}^2 + \nu_{2,3}^2, \tag{5}$$

where $\nu_{1,2}$ is the correction that would be made to the characteristic frequency were there no coupling between the waves $\vec{k}''$ and $\vec{k}'''$, and $\nu_{2,3}$ would be obtained if there were no coupling between $\vec{k}'$ and $\vec{k}''$.

We see from (5) that the presence of coupling with the third wave can either decrease the instability (in the case when the signs of $\nu_{1,2}^2$ and $\nu_{2,3}^2$ are different) and even produce full stabilization, or increase the instability (when $\nu_{1,2}^2 < 0$ and $\nu_{2,3}^2 < 0$. This can occur, for example, at the bifurcation points of the waves $\vec{k}'$ and $\vec{k}'''$).

It is of interest to note that in the process indicated above one of the excited waves can have a frequency higher than the decaying initial wave, although, as is well known, this is impossible in decay into two waves. Such a decay with a "violet" satellite is possible if in (5) we have $\nu_{1,2}^2 < 0$, $\nu_{2,3}^2 > 0$, and $|\nu_{1,2}^2| > |\nu_{2,3}^2|$ (wave vector of "violet satellite $\vec{k}'''$).**

Let us consider in conclusion a concrete case, the decay of an Alfven wave. In a coordinate frame where the X axis is directed along $\vec{H}_0$ and the Z axis is perpendicular to the plane of the vectors $\vec{k}_0$ and $\vec{H}_0$, the approximate values of ν^2 for decays into different wave pairs are

$$\omega_1^2 = k_1^2 V_A^2,\ \omega_2^2 = k_{2x}^2 S^2,\ \nu^2 = \frac{\delta V^2}{16}\frac{V_A^2}{\omega_1\omega_2}\frac{k_{2x}^2 k_{1z}^2 k_{0x} k_{2x}(k_{1\perp}^2 + k_{1x}k_{2x})}{k_{1\perp}^2 k_2^2},$$

$$\omega_1^2 = k_{1x}^2 S^2,\ \omega_2^2 = k_2^2 V_A^2,\ \nu^2 = -\frac{\delta V^2 V_A^2}{16\ \omega_1\omega_2}\frac{k_{1x}^2 k_{1z}^2 k_{0x} k_{1x}(k_{2\perp}^2 + k_{1x}k_{2x})}{k_1^2 k_{2\perp}^2},$$

$$\omega_1^2 = k_{1x}^2 V_A^2,\ \omega_2^2 = k_{2x}^2 S^2,\ \nu^2 = \frac{\delta V^2}{16}\frac{V_A^2}{\omega_1\omega_2}\frac{k_{2x}^4 k_{1y}^2 k_{0x} k_{1x}}{k_{1\perp}^2 k_2^2}, \tag{6}$$

$$\omega_1^2 = k_{1x}^2 S^2,\ \omega_2^2 = k_{2x}^2 V_A^2,\ \nu^2 = -\frac{\delta V^2}{16}\frac{V_A^2}{\omega_1\omega_2}\frac{k_{1x}^4 k_{2y}^2 k_{0x} k_{2x}}{k_1^2 k_{2\perp}^2}.$$

Here $\vec{k}_2 = \vec{k}_0 + \vec{k}_1$, $\omega_2 = \omega_0 + \omega_1$, S is the speed of sound, V_A is the Alfven velocity, and δV pertains to the initial wave. Using (5) and (6) we can write out the values of ν^2 for different wave triplets. We shall show, for example, that the decay of an Alfven wave into a fast and slow sound wave can be stabilized by an Alfven wave:

$$\omega_1 = k_1 V_A, \quad \omega_2 = k_{2x} S, \quad \omega_3 = -k_{3x} V_A, \quad \mathbf{k}_2 = \mathbf{k}_1 + \mathbf{k}_0, \quad \mathbf{k}_3 = \mathbf{k}_2 + \mathbf{k}_0,$$

$$\omega_2 = \omega_1 + \omega_0, \quad \omega_3 = \omega_2 + \omega_0, \tag{7}$$

$$\nu^2 = \frac{\delta V^2}{16} \frac{V_A}{S} \frac{k_{2x}^2}{k_{1\perp}} \left\{ \frac{k_{1z}^2 (k_{1\perp} - k_{1x})}{k_1} - \frac{k_{1\perp}^2}{k_{3\perp}^2} k_{3y}^2 \right\} \frac{k_{0x}}{k_2^2}.$$

The second term in (7) describes the decay of an Alfven wave into slow and fast sound. We see that if $k_{1\perp} > k_{1x}$ and the absolute value of the first term is larger than that of the second, then this decay will be suppressed by an Alfven wave.

[1] V.N.Oraevskii, R.Z.Sagdeev, ZhTF 32, 1291, 1962, Soviet Phys. Tech. Phys. 7, 955, 1963.
[2] A.A.Galeev, V.I.Karpman, JETP 44, 592, 1963, Soviet Phys. JETP 17, 403, 1963.
[3] V.N.Oraevsii, Nuclear Fusion 4, 263, 1964.
[4] V.D.Fedorchenko, V.I.Muratov, B.N.Rutkevich, ibid. 4, 300, 1964.

*And in spite of the fact that the interaction energy of the corresponding waves differs from zero.

** "Violet" satellites appear frequently in experiments on propagation of waves of appreciable amplitude in a plasma, cf. e.g., [4].

CONCERNING ONE POSSIBILITY OF OBSERVING BRANCH POINTS CONNECTED WITH p AND p' POLES

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In connection with papers [1,2] in which it is shown that Regge poles cannot exist as isolated points and must be accompanied by a whole series of moving branch points, the question arises of experimentally observing effects due to branch points in the scattering of high-energy particles.

Branch points connected with the ρ-pole in πN interaction can apparently be observed by investigating neutron polarization in the charge-exchange reaction $\pi^- p \to \pi^0 n$. If this effect does not indeed decrease with the energy [3], then it offers direct proof of the existence of the branch points connected with the ρ-pole [4].

Some possibilities of observing effects connected with branch points in principal poles with positive signature are discussed by V. N. Gribov [5]. We indicate here another method of observing the effects of branch points connected with p and p' poles. It can be shown that if the asymptotic behavior of πN scattering is determined by two isolated poles with positive signature (p and p') and an arbitrary number of poles with negative signature and with accompanying branch points, then the following relation holds:

$$\left[\left(P\frac{d\sigma}{dt}\right)_- + \left(P\frac{d\sigma}{dt}\right)_+ - \left(P\frac{d\sigma}{dt}\right)_{ex}\right] = F(t)\left(\frac{1}{E}\right)^{2-\alpha_p(t)-\alpha_{p'}(t)}, \quad (1) \tag{1}$$