MULTIPLE POLES AND RESONANCE SPLITTING

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The large and continuously increasing number of hadron resonances observed in recent years apparently makes it worth while to attempt a theoretical analysis of certain singularities that arise when two resonant states (appearing in the same reaction) have nearly equal masses. The problem can be formulated with the aid of the following example: Assume that we have started to decrease the mass difference of the ω and ϕ mesons (known boson resonances with identical quantum numbers). Is it theoretically possible to make the mass difference $m_\omega - m_\phi$ smaller than the width of each of the resonances, and if so, what takes place in this case?

We shall use a phenomenological and sufficiently general approach to clarify these questions. Assume that two scalar* unstable particles a and b with nearly equal masses (the analog of "bare" ω and ϕ mesons) are created in some reaction with respective amplitudes M_a and M_b, and then decay in some state j with amplitudes r_a^j and r_b^j (Fig. 1).

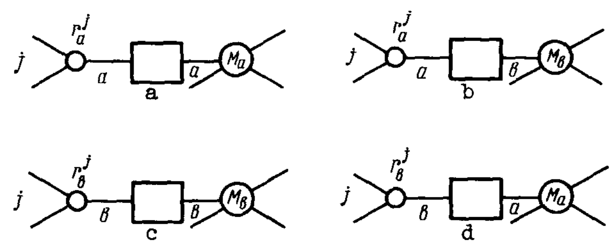

Fig. 1

The diagrams of Fig. 1 take into account only the resonant contribution to the reaction amplitude. The block a–□–a on Fig. 1a is the Green's function $D_a(p^2)$ of particle a, with $D_a^{-1} = p^2 - m^2 - \tilde{\Pi}_{aa}(p^2)$, where p^2 is the square of the invariant mass of the system j. It is usually assumed that the imaginary part and the derivative of the real part of $D^{-1}(p^2)$ can be regarded as constant within the limits of the resonance width (this corresponds to a Breit-Wigner behavior of the resonance amplitude). In our case it is necessary to separate from $\tilde{\Pi}_{aa}(p^2)$ that pole contribution of particle b which varies rapidly just near the resonance. The pole contributions must also be separated from the analogous blocks of Figs. 1b, c, d. This can be conveniently done by using the equations shown graphically in Fig. 2.

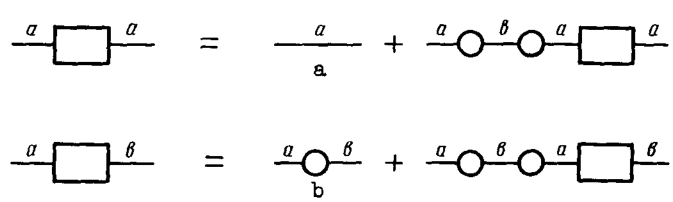

Fig. 2

Similar equations are obtained for the blocks b–□–b and b–□–a by making the substitution a ⇄ b in Fig. 2. In these figures the particle line corresponds to the "propagator" $d_k(p^2) = [p^2 - m_k^2 - \Pi_{kk}(p^2)]^{-1}$, and the block i–○–k corresponds to the expression $d_i \Pi_{ik} d_k$ (i, k = a, b). The irreducible polarization Π_{ik} here no longer contains pole terms, and can be regarded as constant near resonance. The functions M_k and r_k^j (k = a, b), which vary slowly within the resonance width (Fig. 1), can also be regarded as independent of p^2. Using the solution of the algebraic equations on Fig. 2, we obtain for the amplitudes on Fig. 1 the expressions

$$A = r_a^j \, \Delta^{-1} [(p^2 - m^2 - \Pi_b) M_a + \Pi_{ab} M_b] , \qquad (1)$$

$$B = r_b^j \Delta^{-1} [\Pi_{ba} M_a + (p^2 - m^2 - \Pi_a) M_b] \qquad (2)$$

(A - sum of contributions of the diagrams of Figs. 1a,b and B - sum of corresponding contributions on Figs. 1c,d), where $\Delta = (p^2 - m^2 - \Pi_b) - \Pi_{ab}\Pi_{ba}$. In these formulas, $m^2 + \Pi_k = m_k^2 + \Pi_{kk}$ with k = a, b. The total resonance amplitude of the reaction is equal to the sum of all the diagrams of Fig. 1. It can be represented in the form

$$M = A + B = (r_a^j M_a + r_b^j M_b) \frac{\epsilon - a + i\beta}{(\epsilon - \epsilon_o + id_1)(\epsilon + \epsilon_o + id_2)}, \qquad (3)$$

where we have introduced the notation

$$\epsilon = p^2 - m^2 - Re\,\frac{1}{2}(\lambda_+ + \lambda_-),$$

$$\epsilon_o = Re\,\frac{1}{2}(\lambda_+ - \lambda_-),$$

$$d_1 = -Jm\lambda_+ > 0, \qquad d_2 = -Jm\lambda_- > 0,$$

$$-a + i\beta = \frac{i}{2}(d_1 + d_2) + \frac{1}{2}(\Pi_a - \Pi_b)\frac{r_a^j M_a - r_b^j M_b}{r_a^j M_a + r_b^j M_b} +$$

$$+ \Pi_{ab}\frac{r_a^j M_b}{r_a^j M_a + r_b^j M_b} + \Pi_{ba}\frac{r_b^j M_a}{r_a^j M_a + r_b^j M_b},$$

and in turn

$$\lambda_\pm = \frac{1}{2}(\Pi_a + \Pi_b) \pm \frac{1}{2}[(\Pi_a - \Pi_b)^2 + 4\Pi_{ab}\Pi_{ba}]^{1/2}.$$

A very important factor in (3) is the dependence on ε in the numerator.

Expression (3) contains two complex poles with respect to the variable ε, and it is natural to expand it into two terms containing simple poles corresponding to "diagonal" states with definite lifetimes and masses, and which appear usually as Breit-Wigner resonances in the corresponding reaction. On the other hand, if the poles are cloes, i.e., $\varepsilon_0 \ll d_1 \sim d_2$ and $d_1 - d_2 \ll d_1$, then such a diagonalization of the states becomes entirely meaningless, since the residues at the poles are large (and go to infinity as the poles come closer together). In this case it is necessary to analyze the expression for the reaction amplitude in the form (3). The possibility of two close or even multiple poles is not excluded theoretically [1] and can be demonstrated, for example, in the Lie model (see [2]).

Obviously, this closeness of the poles is impossible if any of the main channels of the j decay is forbidden by selection rules for one of the diagonal states, i.e., the residue at the corresponding pole in (3) vanishes rigorously (such a situation is realized for K mesons, where the $K_2 \to 2\pi$ decay is forbidden by virtue of the symmetry properties of the matrix Π_{ik} and the equality of the amplitudes $r_{\bar{K}}^{2\pi} = r_K^{2\pi}$). The partial-width difference connected with this channel leads in this case to a difference in the total widths, and this will prevent coincidence of the poles.

It is easy to verify by analyzing (3) that in the case of two close poles (even coinciding in the limit) the cross section of the reaction, which is proportional to $|M|^2$, has as a rule two maxima of width $\sim d$. The form of the resonance peaks and the distance between them depend strongly, via the parameters α and β on the resonance-production conditions (on the amplitudes M_a and M_b, which are complicated functions of a large number of variables). The positions of the maxima in the cross section is not connected at all, under these conditions within the limits of the width, with the "true" mass of each of the resonances, defined as the real part of the corresponding pole. Splitting of the resonance peak is observed and depends on the creation conditions even if the "true" masses of the two resonant states strictly coin-

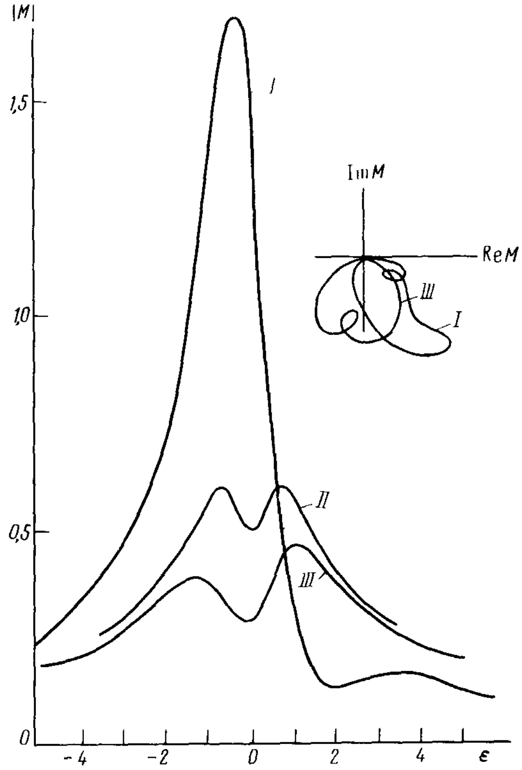

Fig. 3. Dependence of the absolute value of the amplitude (see (3)) on the "energy" ε at different parameter values; I) $\varepsilon_0 = 0$, $d_1 = d_2 = 1$, $\beta = 0.5$, $\alpha = 1.5$; II) $\varepsilon_0 = 0$, $d_1 = d_2 = 1$, $\beta = 0.5$, $\alpha = 0$; III) $\varepsilon_0 = 0.5$, $d_1 = 1$, $d_2 = 1.5$, $\beta = 0.5$, $\alpha = 0$. For the parameter sets I and III we show also the qualitative form of the trajectories of the complex vector M as ε varies from $-\infty$ to ∞.

cide. Figure 3 shows the behavior of the argument and modulus of the amplitude M for certain values of the parameters in formula (3).

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[1] M. Goldberger and K. Watson, Phys. Rev. 136B, 1472 (1964).
[2] J. Bell and C. Goebel, Phys. Rev. 138B, 1198 (1965).

*Allowance for the spin complicates the problem, but in essence does not change the results.