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RADIOPHYSICAL METHODS OF MEASURING THE REST MASS OF THE PHOTON [1)]

M. E. Gertsenshtein and L. G. Solovei
All-union Scientific Research Institute of Physico-technical and Radio-technical Measurements
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Recently I. Yu. Kobzarev and L. B. Okun' [1] and also L. S. Marochnik [2] considered the question of an experimental upper limit of the photon rest mass m_γ. Rocket measurements of the earth's static magnetic field yield a better experimental estimate $\bar{\lambda} = \hbar/m_\gamma c > 0.89 \times 10^5$ km, corresponding to a frequency $\Omega = c/\bar{\lambda} < 3.4$ sec^{-1} $< 4 \times 10^{-48}$ g.

It will be shown in the present paper that radiophysical methods can give a much better estimate of λ. The equations of electrodynamics in the presence of a rest mass are

$$F^{ik}_{,k} - \Lambda^2 A^i = -\frac{4\pi}{c} j^i, \quad F_{ik} = A_{k,i} - A_{i,k}; \quad A^i = (\phi, \mathbf{A})$$
$$F^{ik} = (-\mathbf{E}, \mathbf{H}); \qquad \Lambda = 1/\bar{\lambda} = m_\gamma c/\hbar. \tag{1}$$

The notation is in accordance with [4], and "juggling" of the space indices is connected with a reversal of the sign. When $\Lambda \neq 0$ an additional condition on the potentials $A^i_{,i} = 0$ follows from the continuity equation $j^i_{,i} = 0$ [5] and leads to a wave equation for the vector A^i:

$$A^i_{,i} = 0, \quad F^{ik}_{,k} = -A^i{}_{;k}^{;k} = \Box A^i; \quad \Box = \Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}. \tag{2}$$

We seek a solution of (2) in the form of plane waves $\exp(-ik_i x^i)$, and consider only the longitudinal branch, defined by the condition:

$$k^a \parallel A^{(a)}(\mathbf{k}); \qquad (\Box - \Lambda^2)A^i = 0. \tag{3}$$

Let us consider the nonsingular solution

$$k^a \neq 0, \qquad A^a(\mathbf{k}) \neq 0. \tag{4}$$

Using the relation $A^i_i = 0$, we get

$$A^a_{(\mathbf{k})} = (k^0 k^a/k^2)A^0(\mathbf{k}), \quad \mathbf{E} = i\Lambda^2 A^0(\mathbf{k})(\mathbf{k}/k^2) = i\mathbf{A}(\Lambda^2/k^0), \quad \mathbf{H} = 0. \tag{5}$$

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1) Unscheduled report at GR-5.

The electric field is proportional to Λ^2 and is very small. The smallness of the electric field causes the current produced by the wave propagating in the conducting medium and the damping to be small:

$$(\Box - \Lambda^2)A = -\frac{4\pi\sigma}{c^2}E = -\frac{4\pi}{c}\Lambda^2\frac{i\sigma}{k_0}A, \qquad k^0 = \omega/c,$$
$$k = \frac{\omega}{c}\sqrt{1 - \Lambda^2\left[\frac{1}{k_0^2} - \frac{4\pi\sigma}{k^{03}c}\right]}; \quad \mathrm{Im}\,k \simeq \frac{2\pi\sigma}{c}\left(\frac{\Omega}{\omega}\right)^2. \tag{6}$$

In addition to the waves, there exists also another solution, which is the analog of plasma oscillations:

$$A^0 = 0, \quad A \neq 0, \quad \frac{\partial A}{\partial r} = 0, \quad E = i\omega A \sim e^{-i\Lambda ct} \tag{7}$$

and static solutions. In radiophysical experiments it is customary to use monochromatic oscillations of frequency f, where $\omega = 2\pi f \gg \Omega$, and therefore the static solutions and the solution of (7) are of no interest to us.

It is known that transverse electromagnetic waves do not pass through metallic screens, and their attenuation in screens can be accurately calculated [6]. The passage of longitudinal waves through metallic screens can indeed be used to measure the photon rest mass. Let us consider now the process of interaction of the circuits with the longitudinal waves. If the experiment is performed in the near zone, then it can be described completely with the aid of the interaction Lagrangian

$$L = -\frac{1}{c}\int A_i j^{i*} dv + \text{compl.conj.} = +\frac{1}{c}\int A j^* dv - \int \rho^* \phi\, dv. \tag{8}$$

The potential $\vec{A}$ satisfies the relation

$$\Delta A + k_1^2 A = -\frac{4\pi}{c}j; \quad k_1 = \sqrt{\left(\frac{\omega}{c}\right)^2 - \Lambda^2}$$
$$A = -\frac{1}{k_1^2}\left(\Delta A + \frac{4\pi}{c}j\right) = -\frac{1}{k_1^2}\left\{\mathrm{grad}\,\mathrm{div}\,A + \frac{4\pi}{c}j - \mathrm{rot}\,\mathrm{rot}\,A\right\}. \tag{9}$$

The solenoidal component in (9) should be left out in the presence of metallic screens. Integrating the first term in (8) by parts and leaving out the surface integral, we get

$$L = \frac{1}{ck_1^2}\int \mathrm{div}\,j^*\,\mathrm{div}\,A\,dv - \int \rho^*\phi\,dv - \frac{4\pi}{c^2k_1^2}\int j j^* dv +$$
$$+ \text{c.c.} = L_{int} + L_{self} \tag{10}$$
$$L_{int} = \int\left\{\frac{1}{c^2k_1^2}\frac{\partial\rho^*}{\partial t}\frac{\partial\phi}{\partial t} - \rho^*\phi\right\}dv + \text{кс} =$$
$$= \frac{\Lambda^2}{k_1^2}\int(\rho^*\phi + \rho\phi^*)dv.$$

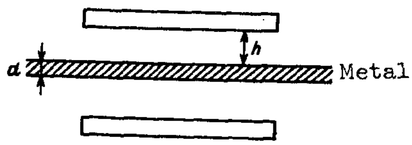

Fig. 1

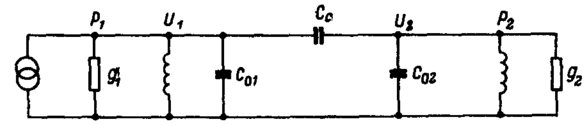

Fig. 2

We have left out here the self-action term

$$L_{self} = \frac{4\pi}{c^2}\int \mathbf{j}\,\mathbf{j}^* dv, \tag{11}$$

which is of no importance in the interaction between two circuits.

Thus, the interaction Lagrangian is proportional to the electrostatic energy of the charges interacting in vacuum without the metallic screens. Therefore, in setting up the equivalent circuit, it is necessary to find the matrix of the mutual capacitance C_{ik} and take it with the factor

$$\left(\frac{\Lambda}{k_1}\right)^2 \simeq \left(\frac{\Lambda}{k^o}\right)^2; \quad L_{ii} = \left(\frac{\Lambda}{k^o}\right)^2 \Sigma C_{ik} U_i U_k \quad , \tag{12}$$

where U_i are the circuit voltages. Let us consider two circuits in which the capacitors have plates on opposite sides of a thin metallic screen (Fig. 1). For the coupling capacitance we get

$$C_C = (\Omega/\omega)^2 C_{geom} = (\Omega/\omega)^2 (S/4\pi d), \qquad h >> d;$$

$$(C_C / C_{01} C_{02}) \simeq (\Omega/\omega)^4 (h/d)^2 ,$$

where S is the area of the plates. The equivalent circuit is shown in Fig. 2, and for the voltage in the second circuit we have at resonance:

$$U_2 = i(\omega C_C\, g_2) U_1 = i\,\Omega^2/\omega^2 (\omega C_{geom} g_2) U_1 \ . \tag{14}$$

It is therefore possible, by using synchronous detection with the generator signal, to measure separately $\mathrm{Re}\,\Lambda^2$ and $\mathrm{Im}\,\Lambda^2$. In the case of weak coupling and in the absence of other capacitances in the circuits, we have for the transfer attenuation

$$P_2 / P_1 = (\omega C_C)^2 / g_1 g_2 = (\Omega/\omega)^4 (C^2_{geom} / C_{01} C_{02}) Q_1 Q_2, \tag{15}$$

where Q is the figure of merit. The power P_2 can be registered if it exceeds the noise level $P_n = kT_n\beta$, where $\beta = \tau/2$ - bandwidth in synchronous detection and τ - observation time. Let us estimate the sensitivity of the method:

$$(\Omega/\omega)_{min} = \sqrt[4]{\frac{kT_n}{2\tau P_1}}(\theta_1\theta_2)^{-1/4}\left(\frac{C^2_{geom}}{C_{01}C_{02}}\right)^{-1/4} . \tag{16}$$

We assume the following values:

$$T_n = 300°K, \quad \tau = 10^6 \text{ sec (10 days)}, \quad P_1 = 40 \text{ W}.$$
$$Q_1 = 100, \quad Q_2 = 100, \quad (C^2_{geom}/C_{01}C_{02}) = 10.$$

This yields $(\Omega/\omega)_{min} = 0.48 \times 10^{-8}$. If the measurements are made at a frequency 100 kHz ($f = 10^5$ Hz), then

$$\Omega < 3.0 \times 10^{-3} \text{ sec}^{-1}, \qquad m_\gamma < 3.5 \times 10^{-51} \text{ g},$$

which is better by approximately three orders of magnitude than in [3]. The values assumed by us are not the limiting ones. A decrease of the working frequency entails considerable technical difficulties.

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CALCULATION OF THE PARAMETERS OF LOW-ENERGY πN SCATTERING

G. M. Radutskii and A. N. Tabachenko
Research Institute of Nuclear Physics, Electronics, and Automation
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Using the premises of current algebra, partial conservation of the axial-vector current (PCAC), and the conservation of the vector current (CVC), the scattering lengths of pions on nucleons were calculated in a number of papers [1,2] in the "soft" pion approximation. This method, however, involves uncertainties connected with extrapolation to the physical pions, so that it is unclear whether the extrapolation procedure or the predictions of the current algebra are actually verified.

A technique was recently developed [3-6] for the calculation of π, ρ, and A_1 vertex functions without using the "soft" pion approximation. The method is based on the assumption that the axial and vector currents satisfy the SU(2) × SU(2) algebra and the PCAC and CVC hypotheses. It is assumed in addition that the vector and axial currents are dominated by mesons with J = 1 and 0. These dynamic assumptions can be expressed in the language of the effective Lagrangians [7], which should be used in lower order of perturbation theory. An advantage of the technique of "hard" pions is the possibility of extending the usual analysis of current algebra to pions with energy much higher than threshold, without using an extrapolation procedure.