We assume the following values:

$$T_n = 300°K, \quad \tau = 10^6 \text{ sec (10 days)}, \quad P_1 = 40 \text{ W}.$$
$$Q_1 = 100, \quad Q_2 = 100, \quad (C^2_{geom}/C_{01}C_{02}) = 10.$$

This yields $(\Omega/\omega)_{min} = 0.48 \times 10^{-8}$. If the measurements are made at a frequency 100 kHz ($f = 10^5$ Hz), then

$$\Omega < 3.0 \times 10^{-3} \text{ sec}^{-1}, \qquad m_\gamma < 3.5 \times 10^{-51} \text{ g},$$

which is better by approximately three orders of magnitude than in [3]. The values assumed by us are not the limiting ones. A decrease of the working frequency entails considerable technical difficulties.

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CALCULATION OF THE PARAMETERS OF LOW-ENERGY πN SCATTERING

G. M. Radutskii and A. N. Tabachenko
Research Institute of Nuclear Physics, Electronics, and Automation
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Using the premises of current algebra, partial conservation of the axial-vector current (PCAC), and the conservation of the vector current (CVC), the scattering lengths of pions on nucleons were calculated in a number of papers [1,2] in the "soft" pion approximation. This method, however, involves uncertainties connected with extrapolation to the physical pions, so that it is unclear whether the extrapolation procedure or the predictions of the current algebra are actually verified.

A technique was recently developed [3-6] for the calculation of π, ρ, and A_1 vertex functions without using the "soft" pion approximation. The method is based on the assumption that the axial and vector currents satisfy the $SU(2) \times SU(2)$ algebra and the PCAC and CVC hypotheses. It is assumed in addition that the vector and axial currents are dominated by mesons with $J = 1$ and 0. These dynamic assumptions can be expressed in the language of the effective Lagrangians [7], which should be used in lower order of perturbation theory. An advantage of the technique of "hard" pions is the possibility of extending the usual analysis of current algebra to pions with energy much higher than threshold, without using an extrapolation procedure.

We shall employ this method in the formalism of [7] to calculate the isotope-antisymmetrical lengths of πN scattering. The πN scattering matrix element is given by

$$<N(p_2)\pi_b(q_2)\text{out}\,|\,N(p_1)\pi_a(q_1)\text{in}> = \\ = iN_{q_1}\int d^4x e^{iq_1x} <N(p_2)\pi_b(q_2)\,|\,i_a(x)\,|\,N(p_1)> , \tag{1}$$

where

$$N_q = \frac{1}{\sqrt{2\omega_q\,(2\pi)^3}}, \\ i_a(x) = \epsilon_{abc}[g_{\pi\pi\rho}v_{\mu b}\phi^{\mu}_c + \partial_\mu(g_{\pi\pi\rho}v^{\mu}_b\phi_c - \lambda_{\pi\pi\rho}\phi_{\lambda b}G^{\lambda\mu}_c \tag{2}$$

φ and v_μ are the field functions of the pion and ρ meson,

$$G^{\lambda\mu}_c = \partial^\lambda v^\mu_c - \partial^\mu v^\lambda_c , \\ \phi^\mu_c = \partial^\mu\phi_c .$$

Substituting in (1) expression (2) for the pion current, we get:

$$T = N_{q_1}N_{q_2}i(2\pi)^4\delta(p_1 + q_1 - p_2 - q_2)i\epsilon_{abc}\frac{(q_1 + q_2)^\mu}{2} \\ \times [2g_{\pi\pi\rho} + \lambda_{\pi\pi\rho}(p_1 - p_2)^2] < N(p_2)\,|\,v_{\mu c}(0)\,|\,N(p_1)> . \tag{3}$$

In the case of scattering of charged pions without charge exchange ($c = 3$) it is possible to use, when writing down the matrix element of the vector current, the model of vector dominance of the electromagnetic current J_μ [8]

$$v_{\mu 3}(0) = g_\rho^{-1}J^V_\mu(0).$$

From (3) we get

$$A^{(-)}(s,t) = g_\rho^{-1}(2g_{\pi\pi\rho} - \lambda_{\pi\pi\rho}t)\frac{F^V_2(t)}{4M}[2(M^2 + \mu^2 - s) - t] , \\ B^{(-)}(s,t) = g_\rho^{-1}(2g_{\pi\pi\rho} - \lambda_{\pi\pi\rho}t)[F^V_1(t) + F^V_2(t)], \tag{4}$$

where [7,9]:

$$g_{\pi\pi\rho} = m^2_\rho g^{-1}_\rho = \frac{1}{\sqrt{2}m_\rho F_\pi}; \qquad \lambda_{\pi\pi\rho} = \frac{\lambda_A}{2g_\rho} \tag{5}$$

and where $F^V_i(t)$ - isovector electromagnetic form factors of the nucleon and $F_\pi = 94$ MeV.

From (4), using the well known formulas (see, e.g., [2]), we calculated the isotope-antisymmetrical S and P scattering lengths and the effective radius

$$
\begin{aligned}
a_{0+}^{(-)} &= \frac{M\mu}{4\pi F_\pi^2 (M+\mu)} F_1^V(0), \\
a_{1+}^{(-)} &= -\frac{M\mu}{3\pi F_\pi^2 (M+\mu)} \left[\frac{F_2^V(0)}{4M\mu} - F_1^{V\prime}(0) + \frac{\lambda_A}{4m_\rho^2} F_1^V(0) \right], \\
a_{1-}^{(-)} - a_{1+}^{(-)} &= \frac{1}{4\pi F_\pi^2 (M+\mu)} \left\{ \frac{\mu}{2M} F_2^V(0) + \left(1 + \frac{\mu}{2M}\right)\left[F_1^V(0) + F_2^V(0)\right] \right\}, \\
R^{(-)} &= \frac{1}{8\pi F_\pi^2 (M+\mu)} \left\{ \frac{1}{\mu}\left(M + \frac{\mu^2}{2M}\right) F_1^V(0) - 4M\mu F_1^{V\prime}(0) - \right. \\
&\quad \left. - \frac{\mu}{M} F_2^V(0) + \lambda_A \frac{M\mu}{m_\rho^2} F_1^V(0) \right\}.
\end{aligned}
\tag{6}
$$

The corresponding numerical values can be compared with those obtained as a result of an analysis of the experimental data given in [10-13].

It is seen from the table that there is a reasonable agreement between our results for the S-wave parameters and experiment. For the P-wave scattering, the agreement with experiment is unsatisfactory. This is connected with the fact that in the existing formulation, the technique of Arnowitt et al. does not make it possible to take into account the contribution from the diagram with baryon exchange, which is important for P-waves.

Authors	$a_{0+}^{(-)}$	$a_{1-}^{(-)}$	$a_{1+}^{(-)}$	$R^{(-)}$
Hamilton, Woolcock	0.086	−0.021	−0.081	0.010
Roper, Wright, Feld	0.085	−0.003	−0.081	—
Samaranayake, Woolcock	0.093	−0.013	−0.081	—
Present work	0.076	0.059	−0.006	0.028

It is precisely the allowance for these contributions in [2,13] which has made it possible, for a definite choice of the parameters, to improve the agreement with experiment for the P-wave scattering lengths.

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SUPERCONDUCTIVITY NEAR A PARAMAGNETIC IMPURITY

A. I. Rusinov
P. N. Lebedev Physics Institute, USSR Academy of Sciences
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It is shown in [1] that introduction of a small amount ($\tilde{<}1\%$) of paramagnetic impurities in a superconductor exerts a strong influence on its properties. In particular, the energy gap of the spectrum of such an alloy no longer coincides with the magnitude of the ordering parameter Δ. The calculation in [1] was carried out in the Born approximation with respect to the impurity potential. It will be shown below that in the case of a superconductor allowance for the total amplitude of the scattering of the electrons by the magnetic impurity leads to a number of interesting results.

We consider first the case of one impurity (placed at the origin) with an interaction in the form

$$\hat{V}(r) = 2\pi/m[U(r) + J(r)\sigma\vec{S}], \tag{1}$$

where $\vec{S}$ - spin of the impurity, which we shall regard as a classical vector. Allowance for the quantum properties of $\vec{S}$ leads to the known Kondo anomaly [2], which arises also in a non-superconducting metal. The foregoing assumption is apparently justified if the spin of the impurity is sufficiently large. The spectrum of the superconductor at $T = 0$, in the presence of an external field $\hat{V}$, is determined by the following system of equations for the coefficients (u, v) of the Bogolyubov transformation [3]:

$$\begin{aligned} \epsilon u_a(r) &= (p^2/2m - \mu)u_a(r) + V_{\alpha\beta}(r)u_\beta(r) + i\Delta(r)\sigma^y_{\alpha\beta}v_\beta(r), \\ \epsilon v_a(r) &= -(p^2/2m - \mu)v_a(r) - V_{\alpha\beta}(r)v_\beta(r) - i\Delta(r)\sigma^y_{\alpha\beta}u_\beta(r) \end{aligned} \tag{2}$$

($\epsilon > 0$). The system (2) breaks up into two independent systems for the pairs (u , v) and (u , v), which are obtained from each other by reversing the sign of J. We choose $\vec{S}$ along the z axis and assume that the parameter Δ does not depend on the coordinates (we shall verify this later). Going over to the Fourier representation and expanding the functions $u(\vec{p})$, $v(\vec{p})$, and $V(\vec{p} - \vec{p}')$ in terms of the harmonics, we get

$$\begin{aligned} u_{\ell\uparrow}(\xi) &= p_0\frac{(U_\ell + J_\ell S)(\epsilon + \xi)\bar{u}_{\ell\uparrow} - \Delta(U_\ell - J_\ell S)v_{\ell\downarrow}}{\epsilon^2 - \Delta^2 - \xi^2}, \\ v_{\ell\downarrow} &= p_0\frac{-(U_\ell - J_\ell S)(\epsilon - \xi)v_{\ell\downarrow} + \Delta(U_\ell + J_\ell S)\bar{u}_{\ell\uparrow}}{\epsilon^2 - \Delta^2 - \xi^2}, \end{aligned} \tag{3}$$

where

$$\bar{f} = \int_{-\infty}^{+\infty} f(\xi)\,d\xi/\pi, \quad \xi = (p^2/2m) - \mu$$