change takes place at distances $r \sim v_F/\tilde{\omega}$ and amounts to $\sim 10^{-4}$.

It is similarly possible to consider the case of two impurities located a distance R apart. It is immediately clear that the arising splitting and shift of the levels of the isolated impurities will be small for $Rp_0 \gg 1$ as a result of the presence of a rapidly oscillating factor $\sin(p_0 R \pm \delta)_{p_0 R}$ in (5). For impurities with antiparallel spins, the energy level remains degenerate and is determined by

$$\bar{\epsilon}_{\uparrow\downarrow} = \epsilon_0 \left[1 + \frac{1}{4} \frac{tg^2 2\delta}{(p_0 R)^2} e^{-(2R/\xi)\sin 2\delta} \right] \tag{7}$$

for pure-exchange isotropic scattering ($\delta^+ - \delta^- = \delta$). The bar denotes averaging over distances $R \gg p_0^{-1}$. For the case of parallel spins, the corresponding expression is

$$\bar{\epsilon}_{\uparrow\uparrow} = \epsilon_0 \left[1 + \frac{1}{4} \frac{\sin^2 2\delta}{(p_0 R)^2} \left(1 + \frac{4R \sin 2\delta}{\xi} \right) e^{-(2R/\xi)\sin 2\delta} \right]. \tag{8}$$

In the latter case, splitting of the levels takes place and has an oscillatory character

$$\epsilon = \epsilon_0 \left[1 \pm tg 2\delta \frac{\sin p_0 R}{p_0 R} e^{-(R/\xi)\sin 2\delta} \right]; \tag{9}$$

the minus (plus) sign pertains to the symmetrical (antisymmetrical) solution relative to the center of the impurity pair. The averaging of the expressions (7) and (8) over the entire sample for finite impurity concentrations leads to the conclusion that the isolated levels cease to exist when the electron mean free path becomes comparable with the coherence length ξ. If the scattering is not too weak, this corresponds to concentrations at which the superconductivity is completely destroyed in the system.

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JOSEPHSON EFFECT IN SUPERCONDUCTING POINT CONTACTS

L. G. Aslamazov and A. I. Larkin
L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
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Radiation from a superconducting point contact was observed in experiments [1] when DC was made to flow through the contact. Exposure of the contact to an external electromagnetic field led to the appearance of steps on its current-voltage characteristic. We show that the point contact of superconducting electrodes has certain properties of a Josephson element; this agrees well with the experimental data.

It is assumed below that the dimension of the contact is small compared with the dimension of the pair ξ and with the depth of penetration of the magnetic field into the superconductor. The influence of the intrinsic magnetic field on the current through the contact can be disregarded. At distances from the contact much smaller than ξ, the ordering parameter Δ varies rapidly with distance. In this region its dependence on the coordinates is obtained from the equation

$$\nabla^2\Delta = 0 \tag{1}$$

where ∇^2 denotes the Laplace operator; on the surface of the superconductor, $\partial\Delta/\partial n = 0$. At distances on the order of or larger than ξ, nonlinear terms are already important. We shall assume that in this region the current density is small compared with the critical one, and therefore Δ is not equal to its value in the bulky superconductor.

We can write the solution of (1) in the form of a sum of two terms

$$\Delta = \Delta_M[f(\mathbf{r})e^{i\chi_1} + (1 - f(\mathbf{r}))e^{i\chi_2}], \tag{2}$$

where $f(\vec{r})$ is the solution of Eq. (1), which tends asymptotically to unity when the distance from the contact increases towards one of the superconductors, and to zero with increasing distance towards the other. The phases χ_1 and χ_2 do not depend on the coordinates, but can depend on the time.

Assuming that the mean free path of the electrons at the contact is much smaller than the contact dimensions, and that the temperature is close to critical, we get for the current density the expression

$$\mathbf{j} = -\sigma\nabla\phi - iC(\Delta\nabla\Delta^* - \Delta^*\nabla\Delta), \tag{3}$$

where the first term is the density of the normal current (σ - conductivity of the metal in the normal state, φ - scalar potential). The constant C was determined in [2].

The potential φ satisfies Eq. (1) and the same boundary condition on the surface of the superconductor as Δ. It is therefore expressed in terms of the function $f(\vec{r})$, namely $\varphi = f(\vec{r})V + \varphi_{-\infty}$, where $V = \varphi_{+\infty} - \varphi_{-\infty}$ is the voltage on the contact. Substituting this expression and expression (2) in formula (3), we get

$$\mathbf{j} = -\nabla f(\mathbf{r})[\sigma V - C\Delta_M^2 \sin(\chi_2 - \chi_1)]. \tag{4}$$

In a superconductor far from the contact, where Δ and φ do not depend on the coordinates, we have $\dot{\chi}_{1,2} = (\hbar/2e)\varphi_{\pm\infty}$. Integrating expression (4) for the current density over the contact cross section, we obtain

$$IR = V + I_K R \sin(2e/\hbar \int V dt), \tag{5}$$

where I is the specific total current through the contact, R the contact resistance in the normal state, and I_c the critical current of the contact, equal to $C\Delta^2/\sigma R$. In the case of a contaminated metal ($l \ll \hbar v_f/T_c$, where l - electron mean free path) we have $RI_c = \pi\Delta^2 eT$.

For a contact having the form of a single-cavity hyperboloid of revolution with a neck dimension a and an aperture angle 2θ, the resistance is $R = \cot(\theta/2)/\sigma a$. At $\theta = \pi/2$, this formula gives the resistance of an aperture of diameter a in the thin layer insulating the two metals.

Equation (5) has the same form as in the case of two superconductors separated by a normal metal [3]. When $I \leq I_c$, it has a solution $V = 0$ and a constant superconducting current flows through the contact. If a constant current $I > I_c$ flows through the contact, then an alternating voltage appears across the contact. Solving (5) we get

$$V = \frac{RI(1-\gamma^2)}{1-\gamma+2\gamma\sin^2\left(\frac{eRI}{\hbar}\sqrt{1-\gamma^2}\,t\right)}; \quad \gamma = \frac{I_c}{I}. \tag{6}$$

It follows from (6) that when γ is close to unity the voltage has a time dependence in the form of narrow periodic pulses of Lorentz shape; if $\gamma \ll 1$, then the voltage varies harmonically.

Expanding (6) in a Fourier series, we get

$$V = \bar{V}[1 + 2\sum_{n=1}^{\infty}\left(\frac{1-\sqrt{1-\gamma^2}}{\gamma}\right)^n \cos n\omega_0 t; \quad \omega_0 = \frac{2e\bar{V}}{\hbar}, \tag{7}$$

where the average voltage across the contact is given by the formula

$$\bar{V} = R\sqrt{I^2 - I_c^2}, \tag{8}$$

which represents the volt-ampere characteristic of the element. The amplitudes of the ac harmonics decrease like a geometric progression.

When the sample is placed in an external electromagnetic field of frequency ω, the total current through the contact becomes alternating. In this case we have in the left side of (5) $I(t) = I_0 + I_1\sin\omega t$, where I_1 is proportional to the amplitude of the external field and is usually small compared with I_0.

The change of the current-voltage characteristic of the contact is of second order in I_1, with the exception of the case $\hbar\omega = 2e\bar{V}$. Let us find the dependence of the average current I_0 on the average contact voltage $\bar{V}$ at a specified amplitude I_1 of the alternating current. The voltage across the contact can be written in the form $V = V_0 + V_1$, where the time dependence of V_0 is given by the formula (6) with an arbitrary phase shift δ relative to $I(t)$, in which the parameter I is connected with $\bar{V}$ by formula (8). $\bar{V}_1$ is equal to zero. The voltage V_1 is proportional to I_1 and is determined from the equation

$$V_1 + \frac{2e}{\hbar}V_c\cos\left(\frac{2e}{\hbar}\int V_0\,dt\right)\int V_1\,dt = RI_1\sin\omega t + RI_0 - \sqrt{\bar{V}^2 + V_c^2}, \tag{9}$$

$$V_c = RI_c.$$

Solving this equation, we obtain from the condition $\bar{V}_1 = 0$ the connection between the average current I_0 and the average voltage $\bar{V}$

$$RI_o = \sqrt{\bar{V}^2 + V_c^2 + \frac{V_c I_1 R}{\sqrt{V^2 + V_c^2}} \sin\left(\frac{2e\bar{V}t}{\hbar} - \omega t - \delta\right)}. \tag{10}$$

The second term of this expression differs from zero only when $\omega = 2e\bar{V}/\hbar$. In this case, at a specified average voltage across the contact, the average current varies as a function of the phase difference δ between the oscillations of the external field and the natural oscillations arising in the contact. In experiment one usually sets the average current through the contact. A step is then obtained on the current-voltage characteristic, with a horizontal section ($\bar{V}$ = const) corresponding to the change of δ from zero to $\pi/2$. In the next higher orders in I_1, steps occur similarly at multiple frequencies.

The obtained relation between the average current flowing through the contact and the phase difference δ can be verified by observing, for example, the interference between the external and intrinsic radiation. At current values corresponding to the step on the current-voltage characteristic, the power of the radiation from a contact excited by an external frequency will increase in proportion to $V_1 \sin\delta$, where V_1 is the first harmonic of the contact voltage, determined from formula (7). We note that this effect is proportional to V_1 and can therefore be observed more readily than the direct radiation from the contact, which is proportional to V_1^2.

In an alternating field, the critical current of the contact decreases. At zero average contact voltage, in second order in I_1, we obtain for the average current

$$I = I_c \sin\chi_0 \left[1 - \frac{(eRI_1)^2}{(\hbar\omega)^2 + e^2 V_c^2 \cos^2\chi_0}\right]. \tag{11}$$

For not too low frequencies ω, the maximum current corresponds to χ_0 close to $\pi/2$.

The described picture of the nonstationary phenomena in superconducting contacts with dimensions $a \ll \xi$ takes place when the current through the contact is not too large: $I \ll I_c \xi/a$. In the case of a contaminated metal, $\xi^2 \sim \hbar l v_F/(T_c - T)$. For very sharp contacts with a small aperture angle θ, it is necessary to replace a by the parameter a/θ. The limitation on the current follows from the condition that at distances on the order of ξ from the point of the contact, where the nonlinear effects are significant, the current density I/ξ^2 should be much smaller than the critical density $C\Delta^2/\xi$. For narrow contacts, this condition is satisfied in a wide range of currents. At large currents and in wide contacts, nonstationary phenomena arise in a region whose dimension is larger than ξ.

In ordinary Josephson elements, the dielectric slab separating the superconductors may contain metallic bridges, each of which is a superconducting contact with $\theta = \pi/2$. The results show that the presence of such bridges does not change qualitatively the picture of the Josephson effect.

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INTERFERENCE OF CONVERSION AND PHOTOEFFECT PROCESSES UPON ABSORPTION OF MOSSBAUER RADIATION

Yu. M. Kagan, A. M. Afanas'ev, and V. K. Voitovetskii
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1. An interesting phenomenon takes place when resonant γ radiation interacts with atoms. We have in mind the fact that two inelastic processes occurring on the same atom - conversion and photoeffect - can interfere with each other. For this to happen it is necessary that the final states be strictly identical in both cases, i.e., that they be physically indistinguishable. It is possible to distinguish between these processes only if the spin of the nucleus changes during the conversion process. Otherwise photoeffect and conversion are indistinguishable.

In principle, interference of these inelastic processes can be observed by measuring the differential cross section of the (γ, e) reaction, or even in ordinary measurements of γ-quantum absorption - in both cases as functions of the velocity of the source relative to the absorber in Mossbauer-type experiments. (The conversion amplitude changes and the amplitude of the photoeffect remains unchanged with changing velocity.)

2. The amplitude of conversion with absorption of a γ quantum having a wave vector $\vec{k}$ and with emission of an electron with momentum $\vec{p}$, accompanied by a transition of the atom from the ground state to the state α, and of the phonon spectrum from the state $\{n_0\}$ to the state $\{n\}$, can be written in the form

$$f_{c_{ii'}} = f^{0}_{c_{ii'}}(\mathbf{k};\ \mathbf{p},\ \alpha)(e^{i\mathbf{ku}})_{\{n_0\}\{n\}}(e^{-i\mathbf{pu}})_{\{n_0\}\{n\}}\,. \tag{1}$$

Here i, i' are the initial and final values of the projection of the ground state of the nucleus and $\vec{u}$ is the displacement of the atom. The index "0" denotes the amplitude corresponding to a rigidly secured nucleus.

For the photoeffect we have analogously

$$f_{ph} = f^{0}_{ph}(\mathbf{k};\ \mathbf{p},\ \alpha)(e^{i(\mathbf{k}-\mathbf{p})\mathbf{u}})_{\{n_0\}\{n\}}\,. \tag{2}$$

For the coherent part (i' = i) of the differential cross section of the process (γ, e) we get