ELECTROMAGNETIC CURRENT AND PARTIAL CONSERVATION OF THE TENSOR CURRENT (PCTC)

G. S. Iroshnikov
Moscow Engineering-physics Institute
Submitted 9 December 1968)
ZhETF Pis. Red. 9, No. 2, 159-162 (20 January 1969)

Recently, an operator relation connecting the electromagnetic current I_μ with the divergence of the tensor current $I_{\mu\nu}$,

$$\partial_\nu T_{\nu\mu}(x) = \frac{m}{\sqrt{2}} I_\mu(x) \tag{1}$$

(m - mass of the vector meson), was proposed and used to solve concrete problems.

The difficulties arising in the analysis of this relation were also pointed out, namely the vanishing of the electric charge [1,2] and the static interaction between the vector mesons and the nucleons [1,5] in the case of a nonsingular behavior of the quantity $\partial_\nu T_{\nu\mu}(x)$ at zero momentum transfer. To overcome these difficulties, Krolikowski [1] proposed that relation (1) should contain an additional term.

Using the electromagnetic current of the $1/2^+$ baryons, we establish below a relation that is free of the indicated difficulties, and discuss the applicability of the PCTC hypothesis [2,5-7] at small momentum transfers.

The electric and magnetic form factors (FF) which determine the matrix element of the electromagnetic current $I_\mu(x)$ between single-particle states of the baryons A and B

$$<B|I_\mu(0)|A> = (1-x)^{-1}\bar{u}(p_B)\left\{\frac{q_\mu}{2M} G_e(k^2) + r_\mu G_m(k^2)\right\} \frac{u(p_A)}{(2\pi)^3}, \tag{2}$$

where

$$k = p_B - p_A, \quad q = p_B + p_A, \quad x = \frac{k^2}{4M^2} \text{ и } r_\mu = \frac{1}{4M}(\hat{q}\hat{k}\gamma_\mu - \gamma_\mu\hat{k}\hat{q}),$$

can always be represented, by virtue of the normalization condition, in the form

$$G(k^2) = G(0) + k^2 f(k^2) \tag{3}$$

if the function $f(k^2)$ is regular in the vicinity of zero.

Inasmuch as the matrix element of the divergence of the tensor current is of the form

$$<B|\partial_\nu T_{\mu\nu}(0)|A> = (1-x)^{-1}\bar{u}(p_B) \frac{q_\mu}{2M} k^2 G_1(k^2) + r_\mu k^2 G_2(k^2) \frac{u(p_A)}{(2\pi)^3}, \tag{4}$$

where the functions G_1 and G_2 are the Sachs FF of the tensor current, determined by the expression

$$< B|T_{\nu\mu}(0)|A> = i(1-x)^{-1}\bar{u}(p_B) \frac{(q_\nu k_\mu - q_\mu k_\nu)}{2M} G_1(k^2) +$$
$$+ (r_\nu k_\mu - k_\nu r_\mu) G_2(k^2) + [r_\nu, r_\mu] G_3(k^2) \frac{u(p_A)}{(2\pi)^3}, \tag{5}$$

the matrix element of the current I_μ can always be connected with the matrix element of a certain tensor current, meaning in terms of the FF:

$$G_{e,m}(k^2) = G_{e,m}(0) + ck^2G_{1,2}(k^2), \tag{6}$$

where c - constant. In operator form, we get from formulas (2), (4), and (6) (the charge e is best set equal to unity)

$$I_\mu(x) = j_\mu(x) + c\partial_\nu T_{\nu\mu}(x), \tag{7}$$

where the current of the free particles $j_\mu(x)$ contains the total magnetic moment of the baryon in accordance with the normalization of the magnetic FF

$$j_\mu(x) = \frac{1}{2}[\bar{\psi}_{in}(x)\gamma_\mu, \psi_{in}(x)] + \frac{\mu_{an}}{2}\partial_\nu[\bar{\psi}_{in}(x)\sigma_{\nu\mu}, \psi_{in}(x)] \tag{8}$$

(for concreteness we have chosen the in-field as the free field). Although Eq. (7) was obtained using only single-particle states, we assume that it is valid also for many-particle states, since it coincides for the off-diagonal matrix element with Eq. (1), which gives reasonable results for such states [3,4].

To find the constant c^s (s - unitary index), with which the divergence of the tensor current enters in Eq. (7) for the electromagnetic current $I_\mu = I_\mu^3 + I_\mu^8/\sqrt{3}$, we take the matrix element between the states of the vector meson and the vacuum

$$\langle V^s|I_\mu^s(x)|0\rangle = c^s\langle V^s|\partial_\nu T_{\nu\mu}(x)|0\rangle. \tag{9}$$

On the mass shell of the meson, the PCTC relation [2,5-7] is satisfied exactly

$$\partial_\nu T_{\nu\mu}^s(x) = \frac{f_\pi}{\sqrt{2}}m_s^2\phi_\mu^s(x), \tag{10}$$

where $f_\pi = \sqrt{2}Mg_A/g_{\pi NN}$ and $\varphi_\mu^s(x)$ - field of vector meson. On the other hand, on the mass shell we have, by definition [8],

$$\langle V^s|I_\mu^s(x)|0\rangle = -\frac{m_s^2}{2\gamma_{Vs}}\phi_\mu^s(x). \tag{11}$$

Comparing expressions (9), (10), and (11), we obtain the constant c^s

$$c^s = -\frac{1}{\sqrt{2}f_\pi\gamma_{Vs}}. \tag{12}$$

This quantity can be simplified by using the Kawarabayashi-Suzuki relation [9]. We then get $|c^s| \sim \sqrt{2}/m_s$.

Equation (7) imposes definite limitations on the FF of the tensor current G_1 and G_2:

1) they must be connected by the relation

$$G_1^p(k^2) = \frac{G_2^p(k^2)}{\mu_p} = \frac{G_2^n(k^2)}{\mu_n}, \tag{13}$$

which is a consequence of the well known relation for nucleon FF [10]; 2) from the dispersion relation for baryon FF, written with one subtraction, it follows that dispersion relations without subtraction are valid for G_1 and G_2.

Equation (7) leads also to a difference in the analytic continuations of PCAC [11] and PCTC in the region of k^2. Whereas for the former such a continuation is apparently satisfactory, for the latter we have a different situation. The matrix element of the divergence (4) vanishes when $k^2 \to 0$. Therefore, at k^2, within the framework of the model of the vector dominance, we obtain another expression

$$\partial_\nu T^s_{\nu\mu}(x) = \frac{f_\pi}{\sqrt{2}} m_s^2 \phi^s_\mu(x) + \sqrt{2} f_\pi \gamma_{Vs} i^s_\mu(x), \tag{14}$$

which goes over as $k^2 \to m_s^2$ into the usual PCTC relation, inasmuch as only terms with φ_μ and divergence have a singularity near the mass of the vector meson.

Finally, the expression for the electromagnetic current of the $1/2^+$ baryons, namely $I_\mu = I^3_\mu + I^8_\mu/\sqrt{3}$, takes the form

$$\begin{aligned} I^s_\mu(x) = i^s_\mu(x) + c^s \partial_\nu T^s_{\nu\mu}(x) = \frac{1}{2}\Big\{ & i f_{slm}[\bar{\psi}^\ell_{in}(x)\gamma_\mu, \psi^m_{in}(x)] + \\ & + \mu_n(i f_{s\ell m} + \frac{3}{2} d_{s\ell m})\partial_\nu[\bar{\psi}^\ell_{in}(x)\sigma_{\nu\mu}, \psi_{in}(x)]\Big\} + c^s\partial_\nu T^s_{\nu\mu}(x), \end{aligned} \tag{15}$$

where the quantities $d_{s\ell m}$ and $f_{s\ell m}$ are the symmetrical and antisymmetrical tensors connected with SU(3) group. In addition, we took account of the fact that

$$\mu^F = \frac{2}{3}\mu^D \approx -\mu_n.$$

Inasmuch as the conserved electromagnetic current is a sum of the currents of all the hadrons, the free currents of all the other strongly interacting particles should, strictly speaking, be added to formula (15).

In conclusion, the author is grateful to V. D. Mur and Yu. P. Nikitin for a discussion of the results.

[1] W. Krolikowski, Nucl. Phys. 87, 650 (1967).
[2] M. Ademollo, R. Gatto, G. Longhi, and G. Veneziano, Phys. Lett. 22, 521 (1967).
[3] L. Maiani and G. Preparata, Nuovo Cimento 48, 550 (1967).
[4] A. Baracca, ibid. 50A, 1010 (1967) and 51A, 560 (1967); A. Baracca, A. Bramon, and A. Tiemblo, ibid. 50A, 1007 (1967).
[5] S. Fubini, G. Segre, and I. D. Walecka, Ann. of Phys. 39, 381 (1966).
[6] W. Krolikowski, Nuovo Cimento 42A, 435 (1966); R. F. Dashen and M. Gell-Mann, Coral Gables Conference, 1966.
[7] G. Costa, C. A. Savoy, and A. H. Zimmerman, Nuovo Cimento 47, 319 (1967).
[8] M. Gell-Mann, Phys. Rev. 125, 1067 (1962).
[9] K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. 16, 255 and 384 (1966).
[10] P. G. O. Freund and R. Oehme, ibid. 14, 1085 (1965); N. N. Bogolyubov, B. V. Struminskii, and A. N. Tavkhelidze, JINR Preprint D-1968, 1965; N. N. Bogolyubov, Nguyen Van Hieu, and B. V. Struminskii, JINR Preprint D-2075, 1965; H. Kleinert, Phys. Rev. 163, 1807 (1967); A. O. Barut, D. Corrigan, and H. Kleinert, Phys. Rev. Lett. 20, 167 (1968).
[11] M. Gell-Mann and M. Levy, Nuovo Cimento 16, 560 (1960).