SPACE-TIME PICTURE OF γ-QUANTUM SCATTERING BY NUCLEONS AT HIGH ENERGIES

B. L. Ioffe
Submitted 10 December 1968
ZhETF Pis. Red. 9, No. 2, 163-166 (20 January 1969)

Some time ago Gribov, Pomeranchuk, and the author [1] raised the question of the distances that play a role in processes of particle scattering at high energies. In the present paper we shall show that in the case of the scattering of virtual γ quanta by nucleons (corresponding to the real electroproduction process) this question can be clarified experimentally, and by the same token, a space-time picture of the scattering process $\gamma + p \to \gamma + p$ can be established at small values of the interval $x_\mu^2 = t^2 - x^2$.

Let us consider the imaginary part of the forward scattering amplitude of a virtual γ quantum with mass $q^2 = q_0^2 - \vec{q}^2$ by a nucleon $M_{\mu\nu}(q_0, q^2)$, where q_0 and $\vec{q}$ are the energy and momentum of the γ quantum [1)]. Strong interactions will be taken into account accurately, and electromagnetic ones in the e^2 approximation. When $q^2 < 0$, the value of $\mathrm{Im}M_{\mu\nu}(q_0, q^2)$ is connected in a well known manner with the total cross section $d^2\sigma/dq^2dq_0$ for the electroproduction of hadrons (see, e.g., [2]):

$$\frac{d^2\sigma}{dq^2dq_0} = \frac{E'_{\!\!,}}{E}\frac{4\pi\alpha^2}{q^4}\left[\cos^2\frac{\theta}{2}W_2(q_0, q^2) + + 2\sin^2\frac{\theta}{2}W_1(q_0, q^2)\right], \tag{1}$$

$$\mathrm{Im}M_{\mu\nu}(q_0, q^2) = p_\mu p_\nu W_2(q_0, q^2) - - \delta_{\mu\nu}W_1(q_0, q^2) + \text{terms proportional to } p_\mu q_\nu,\ p_\nu q_\mu,\ q_\mu q_\nu. \tag{2}$$

Here θ - electron scattering angle, $q_0 = E'$, E and E' - initial and final electron energies, p_μ - 4-momentum of the proton (averaged over the projections of the proton spin). $\mathrm{Im}M_{\mu\nu}(q_0, q^2)$ can be written in the form

$$\mathrm{Im}M_{\mu\nu}(q_0, q^2) = \int d^4x e^{Iqx} A_{\gamma\nu}(x, p), \tag{3}$$

and in the ordinary theory we have

$$A_{\mu\nu}(x, p) = \langle p|[j_\mu(x), J_\nu(0)]|p\rangle. \tag{4}$$

All the calculations will henceforth be carried out in the laboratory system, where $\vec{p} = 0$. We are interested in the region of values of t and x which make an appreciable contribution to the integral (3) at high energies. We note that in the general case the function $A_{\mu\nu}(x, p)$ cannot be determined from data on the electroproduction with the aid of formula (3), inasmuch as the inverse Fourier transformation will give rise to values of $\mathrm{Im}M_{\mu\nu}(q_0, q^2)$ at $q^2 > 0$ in the integral, values which are not obtained in the electroproduction process and arise only in the practically unrealizable process $e^+ + e^- + p \to$ hadrons.

---

1) We indicate only the dependence of $M_{\mu\nu}$ on the invariant variables $pq = mq_0$ and q^2.

Let q_0 be large: $q_0 >> m$ and $q^2 << q_0$[1]. Choosing the z axis in the $\vec{q}$ direction and expanding $|\vec{q}| = \sqrt{q_0^2 - q^2}$ in powers of q^2/q_0^2, we can rewrite (3) in the form

$$\mathrm{Im} M_{\mu\nu}(q_0, q^2) = \int d^4x e^{iq_0(t-z) + i(q^2/2q_0)z} A_{\mu\nu}(x, p). \tag{5}$$

At large values of q_0, an important role is played in the integral of (5) by values $t - z \sim 1/q_0$. Then, as discussed in [1], $A_{\mu\nu}(x, p)$ can behave in two essentially different manners. The first possibility is that large longitudinal distances $z \sim [m_0^2(t - z)]^{-1}$, which increase like $(t - z)^{-1}$, play a role as $t - z \to 0$ (m is a certain effective mass). Such a possibility arises, for example, if the dependence of $A_{\mu\nu}(x, p)$ on x^2 is such that finite values of $x^2 \sim 1/m_0^2$ play a role. The other possibility is that finite values of z (or those growing more slowly than $(t - z)^{-1}$) play a role when $t - z \to 0$.

It is seen directly from (5) that these two possibilities lead to essentially different behaviors of Im $M_{\mu\nu}$ in the region of those values of q_0 and q^2 in which $-q^2/2m_0q_2 << 1$. Then, if the first possibility is realized, we have in this case $(q^2/q_0)z \sim q^2/m_0^2$, and an exponential factor, which oscillates rapidly with q^2 at fixed q_0, arises in (5) in the case of large q^2 and leads to a rapid variation of Im $M_{\mu\nu}$. In the case of the second possibility $(q^2/2q_0)z$ is small when $-q^2/2m_0q_0 << 1$, and consequently Im $M_{\mu\nu}$ does not depend on q^2 in this region of variables.

We thus arrive at the following conclusion. If the invariant functions $w_2(q_0, q^2)$ and $w_1(q_0, q^2)$, which determine the total electroproduction cross section (1), change significantly with variation of q^2 at fixed q_0 when $q_0 >> m_0$ and $-q^2/2m_0q_0 << 1$, then large longitudinal distances, which increase linearly with increasing energy, play an important role in the scattering of gamma quanta by the nucleon. On the other hand, if the functions $w_2(q_0, q^2)$ and $w_1(q_0, q^2)$ remain practically unchanged at fixed q_0 when $q_0 >> m_0$ and $-q^2/2m_0q_0 << 1$, then finite distances (or those increasing more slowly than $(t - z)^{-1}$) play a role in this process[2].

The presently available experimental data [3] indicate that in the region of variables of interest to us, $q_0 >> m$ and $-q^2/2mq_0 << 1$ the function $w_2(q_0, q^2)$ does not depend on q^2 and

$$w_2(q_0, q^2) \approx \mathrm{const}/q_0. \tag{6}$$

Thus the experimental data offer evidence that finite longitudinal distances play a role in the scattering of gamma quanta by nucleons (or, at least, that the role of the terms corresponding to large distances is relatively small).

---

[1] The condition $q^2/q_0^2 << 1$ follows from the kinematics when $-q^2/m^2 >> 1$. In fact, from the equality of $q + p = p_n$, where p_n is the 4-momentum of the hadron state arising in the electroproduction process (i.e., the real intermediate state in (4)), it follows that $2mq_0 - q^2 + p_n^2 - m^2$ or, inasmuch as $p_n^2 \geq m^2$, it follows that $q_0 \geq (1/2)(-q^2/m) >> \sqrt{q^2}$.

[2] Since we do not know the value of m_0, we must consider a sufficiently large region of variation of q^2, on the oreder of several $(\mathrm{GeV}/c)^2$, to be able to distinguish between the two indicated possibilities.

It is of interest to ascertain what behavior of the function $A_{\mu\nu}(x, p)$ (4), i.e., the matrix element of the current commutator, gives rise to the asymptotic dependence (6). By virtue of the causality condition, the values of x^2 in $A_{\mu\nu}(x, p)$ should satisfy the inequality $x^2 = (t - z)(t + z) - \rho^2 > 0$, where ρ is the transverse distance[1]. From this it follows for $t - z \sim 1/q_0$ and finite values of z that the transverse distances ρ tend to zero with increasing energy, $\rho^2 \approx 1/q_0 m_0$. Bearing this in mind and using (3), we can readily find that the asymptotic behavior (6), i.e., Im $M_{\mu\nu} \approx 1/q_0$, corresponds to the following behavior of $A_{\mu\nu}(x, p)$ as $t - z \to 0$:

$$A_{\mu\nu}(x, p) = f_{\mu\nu}(x, p)\delta(x^2), \tag{7}$$

where $f_{\mu\nu}(x, p)$ depends actually only on one invariant variable px = mt.

It is obvious that all the foregoing can be directly applied to the process $\gamma_\mu + p \to \mu$ + hadrons.

I am deeply grateful to V. N. Gribov for valuable discussions and remarks.

[1] V. N. Gribov, B. L. Ioffe, and I. Ya. Pomeranchuk, Yad. Fiz. 2, 768 (1965) [Sov. J. Nuc. Phys. 2, 549 (1966)].
[2] J. Bjorken, International School of Physics "Enrico Fermi," Course 41, Varenna, Italy, 1967.
[3] W. Panofsky, Vienna Conference of High Energy Physics, 1968.

[1] To avoid misunderstandings, we must emphasize that x is the distance between the γ-quantum emission and absorption points, and not between the coordinates of the quantum and the nucleon.