BENDING OF SURFACE AND SELF-FOCUSING OF A LASER BEAM IN A LINEAR MEDIUM

A. V. Kats and V. M. Kontorovich
Institute of Radiophysics and Electronics, Ukrainian Academy of Sciences
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It is well known that the electromagnetic-field momentum component parallel to the surface is not conserved when light passes through the interface between transparent media. This means that a force ("light pressure" p_{lt}) is exerted by the field on the surface of the medium. The light pressure of a bounded laser beam causes the surface of an incompressible liquid to bend, leading to a change of the reflected and refracted beams, and also to the possibility of self-focusing in a linear medium.

Assuming the bending to be small, we write down the equations of motion of the liquid, averaged over the period of the field [1]:

$$\rho\frac{\partial \mathbf{v}}{\partial t} = -\Delta p' + \rho g, \qquad \operatorname{div}\mathbf{v} = 0, \tag{1}$$

and the boundary conditions on the surface $z = \zeta(\vec{r}, t)$, $\vec{r} = (x, y)$:

$$p'^{II} - p'^{I} - \alpha\Delta_\perp\zeta = \overline{\Pi^{I}_{zz}} - \overline{\Pi^{II}_{zz}} \equiv p_{lt}(\mathbf{r}, t); \quad v_z = \frac{\partial\zeta}{\partial t}. \tag{2}$$

Here

$$\Delta_\perp = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad p' = p - \rho\frac{\partial\epsilon}{\partial\rho}\frac{\overline{E^2}}{8\pi};$$

p, ρ, α, $\vec{v}$ - pressure, density, surface-tension coefficient, and velocity of the liquid;

$$\Pi_{ik} = -\frac{1}{4\pi}[\epsilon E_i E_k + H_i H_k - \frac{1}{2}\delta_{ik}(\epsilon E^2 + H^2)]$$

is the Maxwell stress tensor, and the superior bar denotes averaging over the period of the field.

Since a medium with a larger refractive index $n = \sqrt{\varepsilon}$ corresponds to a larger field energy density, the light-pressure force is directed towards the medium with the smaller optical density:

$$p_{lt} = -\frac{\epsilon - 1}{16\pi}\{|E^{II}_{o}|^2 + (\epsilon - 1)|E^{II}_{oz}|^2\} < 0, \quad 2\mathbf{E} = \mathbf{E}_o(\mathbf{r})e^{i(\mathbf{kr} - \omega t)} + \text{c.c.} \tag{3}$$

We thus arrive at a result that is paradoxical at first glance, namely that when a beam is incident from vacuum the liquid surface should bend towards the beam. Actually we have here a complete analogy with the dragging of a dielectric into the region with the larger static field intensity (compare with the deformation of a plate when a beam passes through it, Fig. 1b). A similar conclusion is obtained from a direct calculation of the momentum flux of the photons in the incident, reflected, and refracted beams. It is curious that the same result remains in force also when $\varepsilon \to \infty$ (ideal dielectric mirror)!

From (1) and (2) we get an equation for the bending:

$$\frac{\partial^2\zeta(\mathbf{r}', t)}{\partial t^2} + \frac{1}{2\pi}\int\frac{d\mathbf{r}}{|\mathbf{r}' - \mathbf{r}|}\Delta_\perp\left(\frac{\alpha}{\rho}\Delta_\perp - g\right)\zeta(\mathbf{r}, t) = -\frac{1}{2\pi\rho}\int\frac{d\mathbf{r}}{|\mathbf{r}' - \mathbf{r}|}\Delta_\perp p_{lt}(\mathbf{r}, t). \tag{4}$$

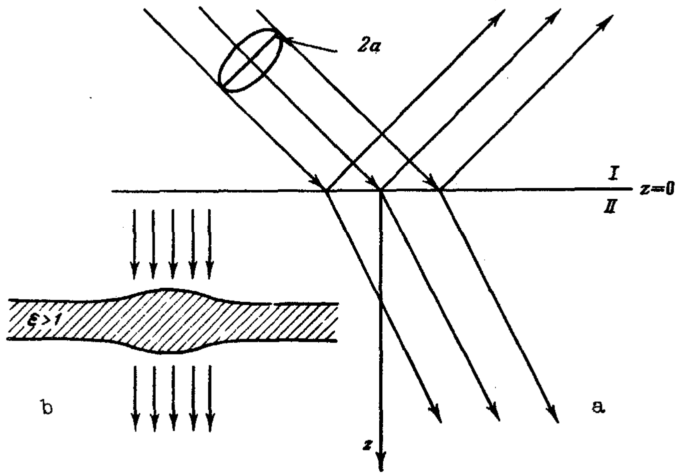

Fig. 1

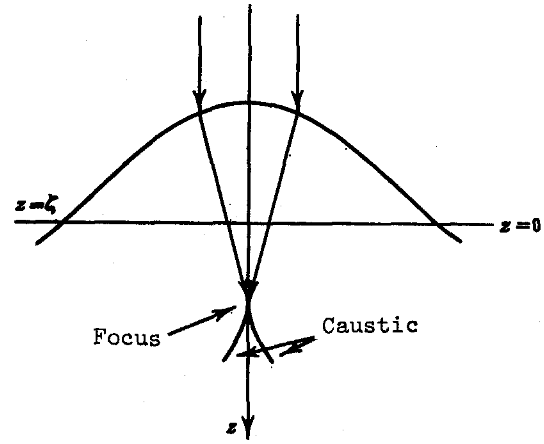

Fig. 2

Under the influence of the laser pulse, which is turned on at t = 0, the surface is set in motion:

$$\zeta(\mathbf{r},t) = \frac{1}{(2\pi)^2\rho}\int_0^t d\tau \int dq\, e^{-iq\mathbf{r}}\, \frac{q\sin\Omega(q)(t-\tau)}{\Omega(q)} \int d\mathbf{r}' e^{iq\mathbf{r}'} p_{1t}(\mathbf{r}',\tau), \tag{5}$$

where $\Omega^2(q) = q(g + \alpha q^2/\rho)$ is the dispersion law of the surface waves. For a rectangular pulse of duration Δ, at small values of the time, $t < \Delta$ and $t \ll \Omega^{-1}(1/a)$ (a - radius of beam) the bending increases quadratically with time. If $p_{1t}(\vec{r}) = -p_0 f(r/a)$, $f(0) = 1$, then:

$$\zeta(\mathbf{r}, t < \Delta) = -\frac{p_0 t^2}{2\rho a}\phi(r/a), \qquad \phi(0) \sim 1. \tag{6}$$

The profile of the bend ϕ is connected with the pressure profile f by

$$\phi(s) = \int_0^\infty I_0(sz) z^2 dz \int_0^\infty I_0(uz) f(u) u\, du. \tag{7}$$

For example, when $f(u) = (1 + u^2)^{-3/2}$ we have $\phi(s) = (2 - s^2)(1 - s^2)^{-3/2}$, and $f(u) = e^{-u^2}$ corresponds to $\phi(s) = \sqrt{\pi}\Phi(3/2, 1, -s^2)$. For the one-dimensional case ($p_{1t} = -p_0 f(x/a)$) we obtain similar formulas, for example $\phi(s) = (1 - s^2)(1 + s^2)^{-2}$ when $f(u) = (1 + u^2)^{-1}$ and $\phi(s) = (2/\sqrt{\pi})\Phi(1, 1/2, -s^2)$ when $f(u) = \exp(-u^2)$. Here $\Phi(\alpha, \beta, \gamma)$ is the confluent hypergeometric function. The stationary value of the bend can also be readily obtained from (5). In order of magnitude we have $\zeta^{stat}_{max} \sim -p_0(\rho g + \alpha/a^2)^{-1}$. When $E_0^2/8\pi \geq (n + 1)\rho\lambda a/(n - 1)t^2$, where λ is the wavelength, the effects connected with the bending exceed the diffraction effects. Knowing $\zeta(\vec{r}, t)$, we can obtain expressions for the fields in the reflected and refracted beams in a linear medium by using the vector equivalent of the Huyghens principle (cf., e. g., [2]). The corresponding formulas are found in [3, 4], which give also the differential characteristics of the wave surfaces (cf., e.g., formulas (4), (5), (10), (13), (17) - (19), and (A.19) in [4]); in our case these characteristics are dependent on the time as a parameter.

The bending of the surface forms a focusing lens, so that in principle the beam can be self-focused in a linear medium (Fig. 2). The central part of the refracted beam is focused within a distance $Z_f(T)$:

$$Z_f^{-1}(t) = \frac{(n-1)^2}{n(n+1)}\,\frac{|E_0|^2}{8\pi}\,\frac{t^2}{\rho a^3}\phi''(0), \quad \phi''(s) = \frac{d^2\phi}{ds^2}. \tag{8}$$

The phenomenon is nonstationary, since the bending, and with it the curvature of the wave front, depend on the time. In a self-focusing nonlinear medium, the curvature of the beam wave front on the boundary, brought about by the bending (curvature $Z_f^{-1}(t)$) leads to motion of the focal spot. The self-focusing distance Z(t) (cf. [6]) equals

$$Z^{-1}(t) = R_{nl}^{-1} + Z_f^{-1}(t), \quad R_{nl} = \frac{a}{2}\sqrt{\frac{\epsilon_0}{\epsilon_2 |E_0|^2}}, \quad \epsilon = \epsilon_0 + \epsilon_2 |E_0|^2. \tag{9}$$

Numerical estimates show that the described effects can apparently be observed in fields of modern powerful lasers. Let, for example, $a = 0.2$ cm, $I = 10$ MW/cm^2, and $\Delta = 10^{-3}$ sec [6]. Then $Z_f(\Delta) \approx 6$ cm at $\rho \sim 1$ g/cm^2, $n \approx 1.3$ and $\zeta(\Delta) \approx 4 \times 10^{-3}$ cm. At an absorption coefficient $\sim 10^{-3} - 10^{-4}$ cm^{-1}, the thermal defocusing leads, according to [7], to effects of the same order of magnitude.

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ANGULAR ANISOTROPY OF Am^{241} NEUTRON-FISSION FRAGMENTS

D. L. Shpak and G. N. Smirenkin

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Only fragmentary information is presently available concerning the angular anisotropy of the fission fragments of odd-odd compound nuclei. We report here the results of detailed measurements of the fragment angular distributions $W(\theta)$ for the reaction $Am^{241}(n, f)$ in a wide range of neutron energies E_n, from 0.3 to 7.2 MeV. The experiment was performed with an electrostatic generator. Glass detectors were used to register the fragments. The measurement procedure was described in detail in [1].

Interest in the study of the angular anisotropy of the $Am^{241}(n, f)$ fission fragments is connected to a considerable degree with the fact that the compound nucleus produced in this reaction is a classical representative of spontaneously-fissioning isomers, or "form isomers."

Before this work was started, the theoretical considerations [2], which are based on the model of the double-hump fission barrier, had already been formulated. According to them,