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QUANTUM KINETIC EQUATION FOR ELECTRONS IN A HIGH-FREQUENCY FIELD

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The general derivation of the kinetic equation [1] is based on the separation of the action of the external field on the electron from the effect of the collisions with the scatterers. It is assumed that the electron moves between collisions under the influence of the field like a classical particle with a dispersion law determined by the band structure of the semiconductor, and that the electron scattering probability does not depend on the field. Accordingly the kinetic equation takes the form

$$\frac{\partial f}{\partial t} + eE\frac{\partial f}{\partial p} = \int \frac{d^3p}{(2\pi)^3}\left[W(p', p)f(p') - W(p, p')f(p)\right], \tag{1}$$

where the right side contains the collision term and the left side the force term. $f(\vec{p})$ is the electron momentum distribution function, $\vec{E}$ is the electric field, which in general depends on the time, and e is the electron charge. $W(\vec{p}', \vec{p})$ is the probability of transition from the state $\vec{p}'$ to the state $\vec{p}$, and is in general the sum of terms of the form

$$|M(p', p)|^2\delta[\epsilon(p') - \epsilon(p) \pm \epsilon_{p',p}], \tag{2}$$

where $M(\vec{p}', \vec{p})$ is the matrix element of the operator of interaction between the electron and the scatterer, $\epsilon(\vec{p})$ is the electron energy, and $\epsilon_{p',p}$ is the energy lost or acquired by the electron on scattering.

The purpose of the present paper is to derive a quantum-kinetic equation for electrons in a homogeneous high-frequency field $\vec{E}(t) = \vec{E}_0\cos\omega t$ under conditions when the field quantum energy $\hbar\omega$ is comparable with the average electron energy $\bar{\epsilon}$.

We derive the equation using an example with electrons having a quadratic dispersion. Defining the electric field in terms of the vector potential $\vec{A}(t) = -(\vec{E}_0 c/\omega)\sin\omega t$, where c is the speed of light, and solving the Schrodinger equation ($\hbar = 1$)

$$i\frac{\partial\psi}{\partial t} = \epsilon(p, t)\psi; \quad \epsilon(p, t) = \frac{\left[p - \frac{e}{c}A(t)\right]^2}{2m}, \tag{3}$$

we obtain the wave function of the electron in the field

$$\psi_p(r, t) = \exp\{ipr - i\int_0^t \epsilon(p, t')dt'\}. \tag{4}$$

Under conditions when $\omega\tau \gg 1$ (τ - free-path time), the canonical momentum $\vec{p}$ is a good quantum number, since it is altered only by the collisions. It is therefore natural to write the kinetic equation for the distribution function of the electrons with respect to the canonical momentum, $F(\vec{p})$ (we use the same letter for the canonical and kinematic momenta, since the corresponding distribution functions are denoted by different symbols).

The amplitude of scattering from the state $\psi_{p'}(\vec{r}, t)$ to the state $\psi_p(\vec{r}, t)$ is given by the integral

$$\int \psi_p^*(r, t)\, H_{int}\, \psi_{p'}(r, t)\, dr dt. \tag{5}$$

We see that inclusion of a homogeneous electric field in $\psi_p(\vec{r}, t)$ does not change the result of integration over the coordinates. The integral with respect to time, which led earlier to an energy conservation law in the form $\delta[\varepsilon(\vec{p}') - \varepsilon(\vec{p}) - \varepsilon_{p',p}]$ now changes to

$$\begin{aligned} &\frac{1}{2\pi}\int_{-\infty}^{\infty} \exp\{-i \int_0^t [\epsilon(p', t') - \epsilon(p, t') \pm \epsilon_{p',p}]dt'\}\, dt = \\ &= \sum_{\ell=-\infty}^{\infty} i^{\ell} I_{\ell}\left(\frac{eE_0(p' - p)}{m\omega^2}\right) \delta[\epsilon(p') - \epsilon(p) \pm \epsilon_{p',p} + \ell\omega]. \end{aligned} \tag{6}$$

In the derivation of (6) we used the relation $\exp[ix \cos\omega t] = \sum_\ell i^\ell I_\ell(x) \exp i\ell\omega t$, where $I_\ell(x)$ is a Bessel function.

It follows from (6) that the quantum-kinetic equation is obtained from (1) by crossing out the term with the high-frequency field in the left side of (1) (since the high-frequency field is fully accounted for in the collision integral) and by replacing expressions of the type (2) by

$$|M(p', p)|^2 \sum_\ell I_\ell^2\left(\frac{eE_0(p' - p)}{m\omega^2}\right) \delta[\epsilon(p') - \epsilon(p) \pm \epsilon_{p',p} + \ell\omega]. \tag{7}$$

We write out the concrete form of the quantum-kinetic equation for the case of electron-phonon scattering

$$\begin{aligned} 0 = 2\pi^2 g^2 \int \frac{d^3k}{(2\pi)^3} |m_k|^2 \sum_\ell I_\ell^2\left(\frac{eE_0 k}{m\omega^2}\right) \Big\{ \Big[(1 + N_k)\, \delta(\epsilon_p - \epsilon_{p+k} + \omega_k + \ell\omega) + \\ + N_k\, \delta(\epsilon_p - \epsilon_{p+k} - \omega_k + \ell\omega) \Big] F(p+k) - \\ - [(1 + N_k)\delta(\epsilon_p - \epsilon_{p+k} - \omega_k + \ell\omega) + N_k\, \delta(\epsilon_p - \epsilon_{p+k} + \omega_k + \ell\omega)]F(p) \Big\}. \end{aligned} \tag{8}$$

The notation is the same as in [2]. In the left side of (8) we can write in the usual manner $(\partial F/\partial t) + e\vec{E}_1(\partial F/\partial \vec{p})$, if $\vec{E}_1$ is a weakly-alternating field. As follows from (8), the quantum-kinetic equation takes explicit account of the possibility of "multiphoton" absorption or emission of an electric field by the electrons of the semiconductor.

We have already mentioned that a criterion of the suitability of Eq. (8) is $\omega\tau \gg 1$. This criterion can be obtained rigorously by deriving formula (8) using the systematic procedure proposed in [2]. A transition to the limit in (8) with $\omega \ll \varepsilon_p$ yields the same equation as obtained after going over in the classical kinetic equation of the type (1) to the canonical momentum with subsequent averaging over the time under the condition $\omega\tau \gg 1$.

For weak fields it is necessary to replace I_0 in (8) by u, put $x/2$ in lieu of $I_1(x)$, and discard the remaining terms of the sum, since at small x we have for the Bessel function $I_\ell(x) \sim 1/\ell! \cdot (x/2)^\ell$. If the argument of the Bessel functio is of the order of unity, all the terms of the series are significant. An estimate of the magnitude of the field yields in this case $\vec{E}_0 \sim e^{-1}(mh^{1/2}\omega^{3/2}$. If we assume $m \sim 0.05 \times 10^{-27}$ g, and $\omega \sim 2 \times 10^{14}$ $\sec^{-1}$ (CO_2 laser), then $\vec{E}_0 \sim 4 \times 10^5$ V/cm, which is large but attainable.

It follows from (8), in particular, that the influence of the high-frequency field on the elastic scattering of the electron (the term with $\ell = 0$ leads to a change of the conductivity of the semiconductor when the latter is exposed to light [3].

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