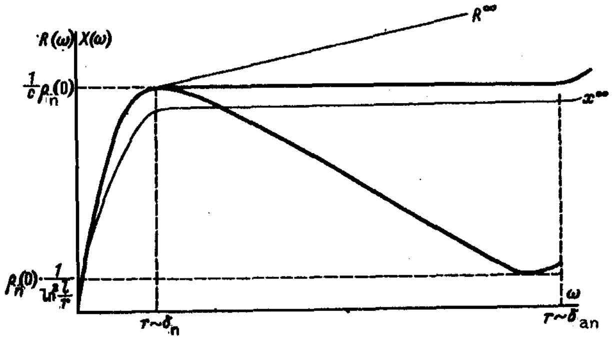

Fig. 1. Real and imaginary parts of the impedance of a half-space vs. frequency. Thin lines - $Z^\infty\delta_n = (c^2/4\pi\omega\sigma_0)^{12}$, $\delta_{an} = (c^2 I/4\pi\omega\sigma_0)^{13}$.

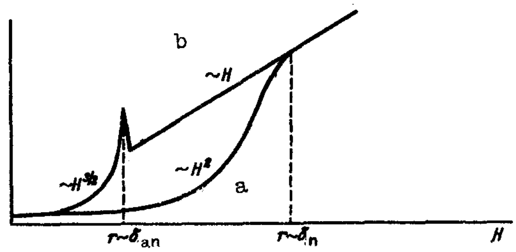

Fig. 2. Impedance vs. magnetic field. a - diffuse scattering, b - case of high symmetry, specular scattering.

with an axis of symmetry not lower than threefold. It is possible to excite here standing waves connected with helicons with a quadratic spectrum [2] or with a spectrum $\omega = (v^4/\Omega^3)k^4$ (the latter exist if the Fermi surfaces of the electrons and holes are similar). To observe the resonances it is necessary to have a plate of thickness $d \ll \ell$; for specular boundaries the heights of the resonances are much higher than for diffuse ones.

In the symmetrical case, near a definite frequency, the impedance of the plate doubles rapidly, as a result of the vanishing of the "surface" current of one of the surfaces.

4. In a quantizing magnetic field the "surface" conductivity also makes a large contribution (if $d \lesssim \ell$) to the total conductivity. The ratio of the amplitude of the Shubnikov - de Haas oscillations to the total conductivity increases on going from diffuse to specular boundaries, as was apparently observed experimentally [3].

A distinguishing freature of the ultraquantum case are strong Sondheimer oscillations (their magnitude relative to the monotonic part of the resistance exceeds the classical value by a factor d/r). Measurement of their period determines the dependence of the number of charges on the magnetic field in the case of overlapping bands (cf. [4]).

StSE plays an important role in other phenomena, too, for example in thermal conductivity and in surface excitation of sound.

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SOUND ABSORPTION IN SUPERFLUID He NEAR THE λ POINT

V. L. Pokrovskii and I. M. Khalatnikov
L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences

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The question of sound absorption near the λ point was considered by Landau and Khalatnikov [1] on the basis of the Landau theory of phase transitions (see [2]). It has been shown experimentally [3] that the Landau thermodynamic theory is not applicable to the case of the λ transition. We shall therefore consider this question again, and show that the result of Landau and Khalatnikov remains valid, in a certain sense, under much broader assumptions.

We assume that the critical phenomena are described by only one characteristic length ξ, which coincides with the correlation radius of the phase of the wave function. This assumption is satisfied in the theory of static and dynamic similarity (see [4, 5]). However, there are also other possibilities of realizing our main assumption. In particular, it agrees with the Landau theory.

The dissipation mechanism consists of transfer of the energy of first sound to the second sound; the latter behaves anomalously near the transition point: its velocity u_2 decreases, and the order of magnitude of the region of the wave vectors k, in which it is defined, equals ξ^{-1}. Ordinary sound (density oscillations) does not play an important role in critical phenomena. It can therefore be expected that the dissipation is determined only by a single characteristic frequency $1/\tau$, which equals u_2/ξ in the case of He II (see [6]).

We shall show that under the foregoing assumption the dependence of $1/\tau$ on $\varepsilon = (T - T_\lambda)/T_\lambda$ is universal, i.e., it contains no critical indices. As in the theory of [1], $1/\tau \sim \varepsilon$. Indeed the quantity $\hbar^2/m^2|\nabla\psi|^2$ is proportional to $\hbar^2\rho_s/m^2\xi^2$, the singular part of the thermodynamic potential ϕ. Therefore $\hbar^2\rho_s/m^2T_\lambda\rho\varepsilon^2\xi^2$ behaves like the singular part of the specific heat C.

We shall use the well known expression for u_2^2 (see [7]): $u_2^2 = \rho_s\sigma^2T/\rho_nC$ (σ - entropy and ρ_n - normal density). The foregoing relations lead to the universal law $1/\tau \sim (mT_\lambda\sigma/\hbar)\varepsilon$. The maximum sound absorption occurs in the frequency region $\omega\tau \sim 1$. We emphasize that first sound exists both at lower and at higher frequencies. It is meaningful to examine it up to frequencies on the order of the interatomic-collision frequency $\nu_a \gg 1/\tau$. Second sound, on the other hand, exists only up to $\omega\tau \sim 1$. This coincides with the estimate $k\xi \sim 1$ assumed in dynamic similarity theory [5]. The kinetic equation describing the approach of a parameter of the order of ψ to the equilibrium state is of the form

$$i\hbar \frac{\partial\psi}{\partial t} = -i\Lambda \frac{\partial\phi}{\partial\rho_s} m\psi + \text{nondissipative terms} \tag{1}$$

(for details see [8]). It is assumed that at low frequencies of motion the quantity $\partial\psi/\partial t$ can be expanded in a series in $\partial\phi/\partial\rho_s$. This is a natural assumption, inasmuch as the point $\omega = 0$ is analytic for any $\varepsilon \neq 0$. The dimensionless kinetic coefficient Λ, which describes the dissipation, should be made up of quantities characterizing the second sound (u_2, ξ) and also of the quantities m and $\hbar$. It is assumed that the attenuation does not depend on the number of particles $n\xi^3$ in a volume with linear dimensions ξ, and consequently it does not depend on $\rho = nm$.[1)] Inasmuch of the dissipation mechanism consists of transferring energy to second-sound quanta, it is natural to assume that $\Lambda \sim u_2$. All the foregoing leads unambiguously to a formula that is valid accurate to a numerical coefficient, $\Lambda = mu_2\xi/\hbar$. In the Landau theory, Λ tends to a constant value at the transition point.

[1)]The dependence on ρ_s is also eliminated, since it is included by assumption in $\partial\phi/\partial\rho_s$.

Let us estimate the relaxation time:

$$\tau^{-1} \sim \Lambda(\partial\phi/\partial\rho_s)(m/\hbar) \sim (mu_2\xi/\hbar)(\hbar^2/m^2\xi^2)(m/\hbar) \sim (u_2/\xi),$$

which agrees with the aforementioned definition of τ.

The propagation of the sound is described by the equations of two-fluid hydrodynamics, supplemented with Eq. (1) (see [7, 8]). The dispersion-equation solution corresponding to first sound is

$$1/u_1^2 = (1/u_0^2) - [(1/u_{10}^2) - (1/u_\infty)]\, i\omega\tau/(1 + i\omega\tau), \tag{2}$$

where $u_1 = \omega/k$, u_{10} is the velocity of the low-frequency sound $[u_{10}^2 = (\partial p/\partial\rho)_\pi$, $u_{1\infty}$ - velocity of high-frequency sound $(1/\tau \ll \omega \ll \nu_a)]$. The difference in the square brackets in formula (2) is equal to

$$(1/u_{10}^2) - (1/u_{1\infty}^2) = [(\rho_s/\rho)_T(\rho_T/\sigma_T) - \rho^2(\rho_s/\rho)_p]^2(\partial^2\phi/\partial\rho_s^2)1/\rho. \tag{3}$$

The time τ in formula (2) equals

$$[2\Lambda m\rho_s/h(\partial^2\phi/\partial\rho_s^2)_{\sigma=\mathrm{const}}]^{-1}.$$

In formula (3), the derivative $\partial^2\phi/\partial\rho_s^2$ is taken at constant entropy σ. Its connection with the isothermal derivative is

$$(\partial^2\phi/\partial\rho_s^2)_{\sigma=\mathrm{const}} = C/C_{\rho s}(\partial^2\phi/\partial\rho_s^2)_{T=\mathrm{const}}.$$

Here $C_{\rho s}$ is the specific heat at the specified value of ρ_s. The subscripts T and p denote differentiation. The independent variables are chosen to be T and p. At any power-law dependence of ϕ on ε, the terms in the square brackets in (3) cancel out in the principal order of magnitude. In the first nonvanishing order in ε, the difference $u_{1\infty}^2 - u_{10}^2 = \varepsilon^\alpha$, where α is the critical index of the specific heat. Thus, when the λ point is approached the difference between $u_{1\infty}$ and u_{10} tends to zero. This is connected with the decrease of the phase volume of the undamped second sound.

When $\omega\tau \ll 1$, the damping of the first sound increases with decreasing ε like $\varepsilon^{-1+\alpha}$, in agreement with the measurements of Barmatz and Rudnick [9]. When $\omega\tau \sim 1$, the absorption has a maximum.

It should be noted that the propagation of first sound does not fall into the similarity pattern in the sence that its frequency is not a homogeneous function of the form $\omega = k^\gamma\phi(k\xi)$. This is not surprising, inasmuch as first sound carries fluctuations of the density and not of the phase.

For second sound, in the low-frequency region $\omega\tau \ll 1$, the dispersion equation is of the form

$$u_2^2 = u_{20}^2[1 + i\omega\tau(\partial^2\phi/\partial\rho_s^2)\rho/\sigma_T((\rho_s/\rho)_T + C/T_\lambda\rho)^2]. \tag{4}$$

We note that the coefficient of $i\omega\tau$ does not depend on ε at any value of the critical index α and its order of magnitude is unity. In the case of second sound $\omega\tau \sim k\xi$, so that Eq. (4) represents the first two terms of the expansion of ω in powers of $k\xi$, as is proposed in the dynamic theory of similarity [5] and is confirmed by experiments of Tyson [6]. In the region

$\omega\tau \lesssim 1$ the equations of hydrodynamics are not applicable to second sound. Formula (4) shows that in this region the imaginary part of the frequency becomes comparable with the real part.

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