$$\frac{1}{\sigma^{II}}\frac{d\sigma^{II}}{d\cos\theta}\Bigg|_{\theta\neq 0,\pi} \leqslant \left(1+\frac{\rho'}{2}\right)\frac{S^{1/2}\ln S}{\pi\sin\theta\sqrt{\gamma}},$$

where ρ' is a constant such that $\sigma^{II} \geq \text{const}\cdot S^{-\rho'}$.

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TRANSVERSALITY OF FIELDS OF VECTOR AND AXIAL-VECTOR MESONS AND HELICITY SYMMETRIES

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We establish in this paper, on the basis of the Lagrangian formalism, the connection between the condition of the transversality of the fields of V and A mesons with helicity symmetries. We show that in theories in which the V and A fields remain transverse also in the interactino, a helicity group arises and moreover the field algebra is satisfied.

We consider the most general relativistically- and P-invariant Lagrangian with dimensionless coupling constants, describing the interaction of a generally arbitrary and different number of fields

$$P^a(0^-),\ \sigma^e(0^+),\ V^i_\mu(1^-),\ A^m_\mu(1^+),\ B^r(1/2^+) \tag{1}$$

and we construct the corresponding equations of motion. We assume, in addition, that

$$\partial_\mu V^i_\mu = 0,\ \ i=1,\dots,n_V;\ \partial_\mu A^m_\mu = 0,\ m=1,\dots,n_A. \tag{2}$$

The subsequent analysis is based on a general remark [1], according to which the equations of motion should not produce excessive limitations[1)] on the number of degrees of freedom of the fields (1). Therefore each term of the independent Lorentz structure in the additional conditions

$$(m_V^2)_{ij}\,\partial_\mu V^j_\mu = R^i(P,\sigma,V,A,B) = 0,$$
$$(m_A^2)_{mn}\,\partial_\mu A^n_\mu = Q^m(P,\sigma,V,A,B) = 0$$

(obtained by taking the 4-divergences of the equations of motions of the V and A fields and replacing in the resultant expression the higher-order derivatives of the fields (1) in accordance with the equations of motion, and then using conditions (2)) should vanish as a result of the coefficient. This yields a number of relations for the mass matrices and the

1)Except for the necessary limitations, such as the transversality conditions (2) and the equations of motion of the fields V_μ and A_μ themselves (at $\mu = 4$).

coupling constants in the initial Lagrangian $L^{(0)}(x)$. Taking them into account, we get

$$
\begin{aligned}
L^{(0)}(x) = & -\frac{1}{4} V_{\mu\nu} V_{\mu\nu} - \frac{1}{2} V_\mu m_V^2 V_\mu - \frac{1}{4} A_{\mu\nu} A_{\mu\nu} - \frac{1}{2} A_\mu m_A^2 A_\mu - \\
& - \bar{B}\gamma_\mu (\partial_\mu - i T_i^{(1)} V_\mu^i - i\gamma_5 T_m^{(2)} A_\mu^m) B - \frac{1}{2} (\partial_\mu - \eta^i V_\mu^i - i\Delta^m A_\mu^m) P \times \\
& \times (\partial_\mu - \eta^i V_\mu^i + i\Delta^n A_\mu^n) P - \frac{1}{2} (\partial_\mu - \xi^i V_\mu^i - i\Delta^m A_\mu^m) \sigma \times \\
& \times (\partial_\mu - \xi^i V_\mu^i + i\Delta^n A_\mu^n) \sigma - (\partial_\mu - \eta^i V_\mu^i) P (\partial_\mu - \Delta^m A_\mu^m) \sigma + \\
& + (\partial_\mu - \xi^i V_\mu^i) \sigma (\partial_\mu - \Delta^m A_\mu^m) P + \bar{B} (G_e^{(1)} \sigma^e + i\gamma_5 G_a^{(2)} P^a) B + \\
& + \Pi_{abcd}^{(1)} P^a P^b P^c P^d + \Pi_{abef}^{(12)} P^a P^b \sigma^e \sigma^f + \Pi_{efgh}^{(2)} \sigma^e \sigma^f \sigma^g \sigma^h - \\
& - \frac{1}{2} P \mu_P^2 P - \frac{1}{2} \sigma \mu_\sigma^2 \sigma,
\end{aligned} \tag{4}
$$

where

$$
\begin{aligned}
V_{\mu\nu}^i &= \partial_\mu V_\nu^i - \partial_\nu V_\mu^i + \alpha_{ijk} V_\mu^j V_\nu^k + \beta_{mn}^i A_\mu^m A_\nu^n, \\
A_{\mu\nu}^m &= \partial_\mu A_\nu^m - \partial_\nu A_\mu^m - \beta_{mn}^i (V_\mu^i A_\nu^n - V_\nu^i A_\mu^n).
\end{aligned}
$$

The relations obtained by this method make it obvious that (as a consequence of the traversality condition (2)) the matrices of the coupling constants α, β, η, ξ, and $T^{(1)}$ form a representation of Lie algebra with structural constants α_{ijk} (the matrices α^i - regular):

$$\alpha_{ijk} = -\alpha_{jik} = \alpha_{jki}, [\alpha^i, \alpha^j] = -\alpha_{ijk} \alpha^k, \tag{5a}$$

$$\beta_{mn}^i = -\beta_{nm}^i, [\beta^i, \beta^j] = \alpha_{ijk} \beta^k, \beta_{mn}^i \beta_{pq}^j + \beta_{qm}^i \beta_{pn}^j + \beta_{nq}^i \beta_{pm}^j = 0 \tag{5b}$$

etc; the mass matrices are proportional to the unit matrices for the irreducible representations, for example for the matrices of the V and A fields

$$[\alpha^i, m_V^2] = 0, \quad [\beta^i, m_A^2] = 0 \quad \beta_{m'n}^l (m_A^2)_{m'm} = \beta_{mn}^{l'} (m_V^2)_{l'l} \tag{5c}$$

and $L^{(0)}(x)$ is invariant against the group of transformation of the fields (1) - in infinitesimal form -

$$\delta_\omega V_\mu^i = \alpha_{ijk} \omega_j V_\mu^k, \quad \delta_\phi V_\mu^i = \beta_{mn}^i \phi_m A_\mu^n, \tag{6a}$$

$$\delta_\omega A_\mu^m = \beta_{mn}^i \omega_i A_\mu^n, \quad \delta_\phi A_\mu^m = \beta_{mn}^i \phi_n V_\mu^i, \tag{6b}$$

$$\delta_\omega P^a = \eta_{ab}^i \omega_i P^b, \quad \delta_\phi P^a = \Delta_{ae}^m \phi_m \sigma^e, \tag{6c}$$

$$\delta_\omega \sigma^e = \xi_{ef}^i \omega_i \sigma^f, \quad \delta_\phi \sigma^e = \Delta_{ae}^m \phi_m P^a, \tag{6d}$$

$$\delta_\omega B = -i T_i^{(1)} \omega_i B, \quad \delta_\phi B = -i\gamma_5 T_m^{(2)} \phi_m B, \tag{6e}$$

where ω_i and ϕ_m are the transformation parameters.

A few remarks are in order.

1) The number of V and A fields can be different in the theory. It is easy to imagine, for example, a situation wherein some of the V fields have no axial partners and can serve as a source of invariance against transformations of the type $B \to e^{i\Lambda} B$ with Λ a constant, or of

higher symmetry, and the other part transforms together with the A fields in accordance with (6a, b), with $\beta^i_{jk} = \alpha_{ijk}$ [1]. A suitable example - $(3 \oplus 1)$ - is the theory of the photon and of weak $W^\pm$ bosons (in which the photon is identified with a certain linear combination of the singlet field and one of the members of the triplet)[2], combined with the $U(3) \otimes U(3)$ nonets of the $1^\pm$ mesons. However, much less trivial realizations are also possible. We shall assume henceforth an equal number of V and A fields.

2) Using the canonical commutators for the fields (1), we can readily obtain from (4) that the fields of the V and A mesons satisfy the commutation relations of the helical field algebra [3]. We note that as a result of commutation of the spatial components V^i_ℓ and A^m_ℓ with each other (ℓ = 1, 2, 3), the obtained algebra cannot be of the type of the once-popular $SU(6) \otimes SU(6)$. It should also be pointed out that within the context of the proposed theory, the "field-current identities" are naturally realized [4].

3) In the Lagrangian (4), the fields of spin 1/2 are massless, the masses of the A and V fields are equal (see (5)), and 0^+ fields are present. The next step is to eliminate the 0^+ fields (or at least part of them) from the theory, in analogy with the procedure used in the nonlinear σ model [5]. This leads to the appearance of a mass in the 1/2 fields, and to a mass shift of the V and A mesons, as well as to other consequences of the helicity dynamics [6] discussed recently by many authors (see references in [6]). The theory obtained in this manner can obviously regarded as a minimal helicity dynamics. It proposes a satisfactory description of the low-energy strong interactions (PB decays, $A \rightarrow VP$ decays, etc) and is parametrized, as can be seen from (4) by only two coupling constants: the self-action constant g of the V fields and the constant G of the interaction between the 0^- fields and the 1/2 fields.

Nonminimal theories arise when account is taken in the expansion $L(\ell) = \sum \ell^n L^{(n)}(x)$ (expansion of the "total Lagrangian" in a "fundamental constant" with dimension of length) also of the terms with n = 1,[2] n = 2, etc. It is interesting to note that in this case a certain "economy" in interaction constants is observed: factorization relations between the coupling constants, similar to those arising in the simple pole models with P, V, and A exchange, arise also for the vertices of the described reactions with production of V and (or) A mesons.

A detailed analysis of these questions, as well as questions connected with violation of helicity symmetries, will be presented elsewhere.

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POSSIBILITY OF DETECTING THE W BOSON BY MEANS OF THE POLARIZATION OF THE MUONS FROM THE $W \to \mu + \nu$ DECAY

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The most reliable information on the W boson is presently obtained from neutrino-beam experiments [1,2]. It follows from these experiments that if the W boson exists its mass is $m_W > 2$ GeV.

Besides experiments in a neutrino beam, experiments were made to observe the W boson in NN interactions [3, 4] (of the $NN \to NNW$ type), consisting of searches for high-energy muons emitted at large angles to the proton beam.

The search for the W boson in nucleon-nucleon interactions offer a number of advantages over experiments in a neutrino beam: larger value of the expected cross section [5], much higher intensity and energy of the proton beam compared with the neutrino beam. However, the very difficult background conditions, connected with the presence of muons from the $\pi \to \mu$ decay, make such experiments and their interpretation difficult. Therefore the upper limit obtained in [3] for the W production cross section, $\sigma_W < 2 \times 10^{-34}$ cm^2, is subject to an appreciable uncertainty connected with the estimates of the background due to the $\pi \to \mu$ decay.

In this paper we wish to call attention to an entirely different method of searching for W among the proton-nuclear reaction products.

The point is that the longitudinal polarization of the muons from the $W \to \mu\nu$ decay should have a sign opposite to that of the muons from the decays $\pi \to \mu + \nu$ and $K \to \mu + \nu$. The formula for the longitudinal polarization of $\mu^\pm$ in the $W^\pm$ system in the case of an arbitrary spin state of $W^\pm$ is

$$P(\mu^\pm) = \pm\frac{1 + (3/2)(\vec{\xi}\mathbf{n}) + (3/4)c_{nn} - (\mu^2/2m_W^2)(1 - (3/2)c_{nn})}{1 + (3/2)(\vec{\xi}\mathbf{n}) + (3/4)c_{nn} + (\mu^2/2m_W^2)(1 - (3/2)c_{nn})}, \qquad (1)$$

where $\vec{\xi}$ is the W polarization, $c_{nn} = c_{ab}n_a n_b$, c_{ab} is the W alignment tensor, $\vec{n}$ a unit vector in the momentum direction, μ the muon mass, and m_W the W-boson mass. The sign corresponds to the sign of the charge of $\mu^\pm$. It is seen from formula (1) that $P(\mu^\pm) = \pm 1$ practically always, with the exception of the exotic situation when the following conditions are satisfied: 1) $(\vec{\xi}\cdot\vec{n})$ is close to zero with accuracy $\sim\mu^2/m_W^2$, 2) $c_{nn} = -4/3$ with the same accuracy. The magnitude of c_{nn} for different W-production processes is given by the formula

$$c_{nn} = \frac{2}{3} - 2\frac{|\mathbf{Mn}|^2}{|\mathbf{M}|^2}, \qquad (2)$$