taking the masses of all the other particles from the tables [4], we find that the χ-meson mass is 1400 MeV,** and that of the ρ meson is 790 MeV. It follows therefore that formulas (A) and (B) are well satisfied if one takes the ninth pseudoscalar particle to be the E(1420) meson, whose most likely spin-parity value, according to new experimental data [5], is 0^+. The mixing angle of the η E mesons is 6.5°.

The meson $X^0(960)$ does not fit in the 0^- nonet from $SU_W(6)$. We note that it cannot be definitely assumed that the X^0 meson is pseudoscalar [7]. Its spin-parity was determined in [8] from the decay $X^0 \to \rho + \gamma$ by using only the simplest matrix elements and disregarding the higher multipole transitions, a procedure not convincingly justified at high energy release (∿ 200 MeV). If these transitions are taken into account, then an angular correlation of the $\sin^2\theta$ type appears not only for spin-parity 0^-, but also for 1^+, 2^-, etc. The Dalitz diagram for the $X^0 \to \eta 2\pi$ decay does not make it possible to distinguish between 0^- and 2^- [9] even when the simplest matrix elements are used.

The usual mass formulas of static SU(6) symmetry are obtained for the baryon 56-plet.

In conclusion, we emphasize once more that from the point of view of broken $SU_W(6)$ and $SU_x(6)$ symmetries it is very important to set up experiments aimed at a unique determination of the spins and parities of the E(1420) and $X^0(960)$ mesons. If the experiments confirm the value 0^- for the E(1420) meson, then this meson should be regarded as the ninth pseudoscalar meson within the framework of $SU_W(6)$ and SU_x; failure will serve as weighty evidence against $SU_W(6)$.

The authors are cordially grateful to M. Bendar, B. N. Valuev, M. A. Markov, and especially I. V. Polubarinov for useful discussions.

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*Octet $SU_W(6)$ symmetry breaking of a particular type was considered for vertices and annihilation by Gupta in [3].

**We note that Schwinger's formula [6] yields for the ninth pseudoscalar meson a mass value ∿ 1.6 GeV.

REAL PART OF SCATTERING AMPLITUDE AT HIGH ENERGIES AND DISPERSION SUM RULES

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Submitted 18 June 1967
ZhETF Pis'ma 6, No. 4, 606-608 (15 August 1967)

1. The real part Re f(E) of the πN scattering amplitude at the (presently) maximum attainable energies, in the interval 6 - 28 GeV, was experimentally determined in a number of recent investigations [1,2]. It has turned out that Re f(E) is anomalously large in this

interval, compared with the values expected from different theoretical models and estimates [2,3]. In this connection, sum rules were proposed in [4] for Re f(E), with which to verify the basic premises of the contemporary theory.

We offer below, on the basis of [5-7], new dispersion sum rules that contain information on both Re f(E) and Im f(E). These relations have the advantage that they use practically all the presently known experimental data on the scattering amplitude, but unlike [4] they contain no information on f(o), which is not determined by direct experiment.

2. Let f(E) be the crossing-even combination, usually considered in the investigation of πN scattering, of the zero-angle $\pi^{\pm}N$ scattering amplitudes:

$$f(E) = \frac{1}{2}[f_{+}(E) + f_{-}(E)]. \tag{1}$$

The crossing-symmetry condition yields for real E

$$f^{*}(-E) = f(E). \tag{2}$$

On the basis of well-known results (Greenberg-Low) the following asymptotic inequality certainly holds as $E \to \infty$:

$$|f(E)| < A|E|^{2}; \quad A > 0. \tag{3}$$

Using the method proposed in [5-7] we can derive, from the condition for the analyticity of f(E) and from an additional and physically perfectly natural assumption that the scattering amplitude f(E) is bounded for finite values $|E_k| < \infty$, by taking (2) and (3) into account, the following sought-for dispersion sum rule:

$$\begin{aligned}
&\int_{E_2}^{E_3}[(E^2-E_0^2)(E^2-E_1^2)(E^2-E_2^2)(E_3^2-E^2)]^{-1/2}\,\mathrm{Re}\,f(E)\,E\,dE = \\
&= \int_{0}^{E_0}[(E_0^2-E^2)(E_1^2-E^2)(E_2^2-E^2)(E_3^2-E^2)]^{-1/2}\,\mathrm{Im}\,f(E)\,E\,dE + \\
&+ \int_{E_0}^{E_1}[(E^2-E_0^2)(E_1^2-E^2)(E_2^2-E^2)(E_3^2-E^2)]^{-1/2}\,\mathrm{Re}\,f(E)\,E\,dE - \\
&- \int_{E_1}^{E_2}[(E^2-E_0^2)(E^2-E_1^2)(E_2^2-E^2)(E_3^2-E^2)]^{-1/2}\,\mathrm{Im}\,f(E)\,E\,dE + \\
&+ \int_{E_3}^{\infty}[(E^2-E_0^2)(E^2-E_1^2)(E^2-E_2^2)(E^2-E_3^2)]^{-1/2}\,\mathrm{Im}\,f(E)\,E\,dE.
\end{aligned} \tag{4}$$

Here $0 < E_0 < E_1 < E_2 < E_3 < \infty$ are certain finite values of the energy.

Choosing $E_0 = m$, i.e., equal to the threshold of the physical region, $E_1 \approx 0.3 - 0.5$ GeV, i.e., such that $[E_0, E_1]$ is the low-energy region in which Re f(E) is well known from phase-shift analysis, and E_2 and E_3 respectively the beginning ($E_2 \approx 6 = 26$ GeV) and the end ($E_3 \approx 7 - 28$ GeV) of the high-energy region in which it is important to determine the behavior of Re f(E), we obtain from (4) the sought-for dispersion sum rules. Since Im f(E) $\geq$ 0, Eq. (4) certainly leads to the inequality

$$\begin{aligned}
&\int_{E_2}^{E_3}[(E^2-m^2)(E^2-E_1^2)(E^2-E_2^2)(E_3^2-E^2)]^{-1/2}\,\mathrm{Re}\,f(E)\,E\,dE \geq \\
&\geq \left[\left(m^2-\frac{m^4}{4M^2}\right)\left(E_1^2-\frac{m^4}{4M^2}\right)\left(E_2^2-\frac{m^4}{4M^2}\right)\left(E_3^2-\frac{m^4}{4M^2}\right)\right]^{-1/2} 2m^2 f^2\left(1-\frac{m^2}{4M^2}\right) +
\end{aligned}$$

$$
\begin{aligned}
&+\int_{m}^{E_1}[(E^2-m^2)(E_1^2-E^2)(E_2^2-E^2)(E_3^2-E^2)]^{-1/2}\,\mathrm{Re}\,f(E)\,E\,dE\,-\\
&-\frac{1}{2}\int_{E_1}^{E_2}[(E^2-E_1^2)(E_2^2-E^2)(E_3^2-E^2)]^{-1/2}[\sigma_n^+(E)+\sigma_n^-(E)]\,E\,dE\,+\\
&+\frac{1}{2}\int_{E_3}^{E_{max}}[(E^2-E_1^2)(E^2-E_2^2)(E^2-E_3^2)]^{-1/2}[\sigma_n^+(E)+\sigma_n^-(E)]\,E\,dE.
\end{aligned}
\tag{5}
$$

In (5), $f^2/4\pi \approx 0.08$ is the interaction constant, m and M the pion and proton masses, $\sigma_n^{\pm}(E)$ the total $\pi^{\pm}p$ scattering cross sections, and E_{max} the maximum energy for which $\sigma_n^{\pm}(E)$ is presently known.

It follows from (5), without any calculations, that Re f(E) cannot be too large a negative quantity (Re f(E) ≈ -cE for E = (7 - 12) GeV; $c \approx 1/40\pi(\sigma_n^+ + \sigma_n^-)$), for then (5) would be patently violated. This deduction is analogous to that obtained in [4].

If we choose E_2 in (4) and (5) equal to $E_2 \approx 30$ GeV and $E_3 \approx 30 - 70$ GeV, we get on the basis of (5):

$$
\begin{aligned}
&\int_{E_2}^{E_3}[(E^2-m^2)(E^2-E_1^2)(E^2-E_2^2)(E_3^2-E^2)]^{-1/2}\,\mathrm{Re}\,f(E)\,E\,dE >\\
&>[(m^2-\frac{m^4}{4M^2})(E_1^2-\frac{m^4}{4M^2})(E_2^2-\frac{m^4}{4M^2})(E_3^2-\frac{m^4}{4M^2})]^{-1/2}\,2m^2f^2(1-\frac{m^2}{4M^2})+\\
&+\int_{m}^{E_1}[(E^2-m^2)(E_1^2-E^2)(E_2^2-E^2)(E_3^2-E^2)]^{-1/2}\,\mathrm{Re}\,f(E)\,E\,dE\,-\\
&-\frac{1}{2}\int_{E_1}^{E_2}[(E^2-E_1^2)(E_2^2-E^2)(E_3^2-E^2)]^{-1/2}[\sigma_n^+(E)+\sigma_n^-(E)]\,E\,dE.
\end{aligned}
\tag{6}
$$

This makes it possible to estimate Re f(E) in the energy interval 30 - 70 GeV. On the other hand, by choosing in (5) $E_3 \approx 30$ GeV and $E_{max} \approx 30$ - GeV, we can also estimate Im f(E) in the same interval. A detailed quantitative investigation of the obtained dispersion sum rule will be published separately.

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