we can disregard scattering in the effective region, and the radiation spectrum takes the form

$$\frac{1}{T}\frac{dE}{d\omega}=\frac{e^2}{\pi}\frac{E^2E_s^2}{m^4 aL_{\rm rad}}\frac{1}{\omega}. \qquad (6)$$

In the opposite limiting case

$$\omega << E_s E^2 m^{-3}(aL_{\rm rad})^{-1/2} \qquad (7)$$

the radiation spectrum changes noticeably:

$$\frac{1}{T}\frac{dE}{d\omega}=\frac{2e^2}{\sqrt{3}\pi}E^{2/3}(E_s^2\, 3aL_{\rm rad})^{-1/3}\omega^{1/3}. \qquad (8)$$

It is interesting to note that for entry directions corresponding to the minimum radiation, the intensity remains unchanged, owing to the small number of atoms in the effective region; at the same time, for an entry angle corresponding to maximum radiation, the radiation intensity is noticeably decreased by the scattering. The effect under consideration thus leads to a smoothing of the interference character of the spectrum.

In conclusions, the authors take the opportunity to thank V. M. Galitskii, M. L. Ter-Mikaelyan, and E. L. Feinberg for valuable remarks.

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QUANTUM ELECTRODYNAMICS WITH TWO FERMIONS. II

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It was shown in an earlier paper [1] that in quantum electrodynamics with two fermions (electron and muon), diverging diagrams containing closed loop can cancel each other, and it is then possible to construct a theory free of infinite renormalizations. The proposed modification of the theory met with sharp criticism, since reversal of the sign of the loop reverses also the sign of its imaginary part, which is incompatible with the physical requirements of unitarity. An attempt was made in [1] to get around this objection by using the fact that the imaginary part of the muon loop is smaller than that of the electron loop at all values of photon momentum. But this does not get rid of the objections, since it contradicts the analyticity conditions as applied to individual vertices of the loop. In addition, if a closed muon line has four vertices, then it is connected with the probability of the radiative deceleration and is therefore positive-definite.

We shall attempt here to modify the procedure proposed in [1] in a way as to retain the main result, but avoid any contradiction to the general requirements of field theory. We shall assume that time enters in the muon field with a sign opposite to that in the electron field. Namely, if the electron field operator is (see [2])

$$\psi_e = \Sigma(a_e(p)\,u(p)\,e^{-i\omega_e t + ipr} + b_e^+(p)v(-p)e^{i\omega_e t - ipr}), \tag{1}$$

then the corresponding muon operator is

$$\psi_\mu = \Sigma(a_\mu(p)u'(p)e^{i\omega_\mu t + ipr} + b_\mu^+(p)v'(-p)\,e^{-i\omega_\mu t - ipr}). \tag{1'}$$

The muon creation and annihilation operators satisfy the same anticommutation relations as the electron operators, and $\omega_{e,\mu}$ denotes the positive square root, $\omega_{e,\mu}=\sqrt{m_{e,\mu}^2 + p^2}$. The spinor amplitudes u'(p) and v'(-p) are subjected to an additional Racah transformation $R = \gamma_4\gamma_5$ compared with the amplitudes u(p) and v(-p). Because of this, the time derivative enters with the same sign in the Dirac equations for the electrons and muons, as is required for the invariance of both equations with respect to the gauge of the potentials of the electromagnetic field.

The contraction $<\psi(x)\bar{\psi}(x')>_0$ will consequently take on different forms for the electrons and muons:

$$<\psi_e(x)\,\bar{\psi}_e(x')>_0 = -(\hat{\nabla} - m_e)\int \frac{dp}{2\omega_e} e^{-i\omega_e(t-t') + i(p-p')r}, \tag{2}$$

$$<\psi_\mu(x)\,\bar{\psi}_\mu(x')>_0 = -(\hat{\nabla} - m_\mu)\int \frac{dp}{2\omega_\mu} e^{+i\omega_\mu(t-t') + i(p-p')r}. \tag{2'}$$

The function $(2\omega_{e,\mu})\,\exp[-1 \pm i\omega_{e,\mu}(t - t')]$ must be represented in the form of a contour integral in p_0. Then

$$e^{-i\omega_e(t-t')}(2\,\omega_e)^{-1} = (2\pi i)^{-1}\int e^{-ip_0(t-t')}(p^2-p_0^2+m_e^2-i\delta)dp_0, \tag{3}$$

$$e^{+i\omega_\mu(t-t')}\,(2\,\omega_\mu)^{-1} = (2\pi i)^{-1}\int e^{-ip_0(t-t')}(p^2-p_0^2+m_\mu^2+i\delta)dp_0, \tag{3'}$$

and it is necessary to make a small positive addition to the muon mass in order to obtain the required sign in the exponent on the left side. By the same token, the energy of the free muon is assumed negative. This, however, does not lead to absurd results, since the sign of the energy in the delta-functions of the matrix elements of the muons is opposite to the sign for the photons and electrons. As a result, the energy conservation law is not changed at all by the muon transitions.

At the same time, the effective cross section for the scattering of polarized muons by a polarized target is sensitive to the transformation R. Namely, the scattering matrix element contains the operator $(1 + \gamma_5\hat{S})$, where S is defined in terms of the unit polarization vector in the proper reference frame ξ by means of the formulas $S = \xi + p\xi(p)m^{-1}(E + m)^{-1}$ and $S_0 = \xi(p)m^{-1}$. But the transformation $R^{-1}(1 + \gamma_5\hat{S})R$ reverses the sign of the spatial part of $\gamma_5\hat{S}$). Consequently, if the wave function of the other partner of the collision is not subject to the transformation R, the effective cross section will have a different dependence on the components of the dyad $\xi_{1\ell}\xi_{2k}$ than in the case of particles of the same type.

The results of the theory of muon beta decay are very sensitive to the form of the spinor amplitudes. It is easy to see that if the muonic neutrino is of the same type as the muon under our assumptions, meaning that it depends on the time with the sign reversed and is subject to the transformation R, then all the formulas of the usual theory remain in force (see [3]). We thus have some indication that the different natures of the electronic and muonic neutrinos can be explained.*

The time, which enters in the equation for the scattering matrix, can be assumed arbitrarily to coincide with the electron time. Then the Feynman diagram of the muon loop differs from that of the electron loop in the sign of the infinitesimally small imaginary addition to the mass:

$$I_{e,\mu} = i\int_0^1 dx \int dp \int_{-\infty}^{\infty} dp_0 [p^2 - p_0^2 + m_{e,\mu} \mp i\xi - k^2 x(1-x)]^{-2}. \qquad (4)$$

It is easy to see that the imaginary part of $I_{e,\mu}$ is always finite and does not depend on the sign of ε, whereas the real part, which diverges logarithmically, is determined by the sign of ε. The divergences of the electron and muon loops cancel each other, as required.

There is an experimental possibility of verifying the sign of the muonic polarization of vacuum. The Lamb shift of a heavy mu-mesic atom, if measured with sufficient accuracy, is sensitive to the muonic polarization of the vacuum (in a light atom, the electronic part of the polarization is too overwhelming). But since the muon moves in a heavy mu-mesic atom predominantly inside the nucleus, it is necessary first to sound-out the distribution of the potential in the nucleus with the aid of fast electrons. In an ideally spherical nucleus (Pb^{208}) the non-static fluctuating part of the electromagnetic field inside the nucleus should be small compared with the static field, and should not distort the Lamb shift by itself.

It is quite possible, however, that if it is found that particles other than electrons make a contribution to the vacuum polarization, then a noticeable role will be assumed also by nuclear-active particles, especially pions. Then pure electrodynamics with two fermions, an electron and a muon, even when free of divergences, will not agree with experiment. It is then difficult to expect it to yield the correct ratio of the fermion masses, which was found in [1] to be on the order of 10.

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*See [3], p. 72, third formula from top. The vector S contained in it must be referred to the rest frame, where $S = \xi$. The reversal of the sign means nothing here. But then the result does not depend on the reference frame. A deviation from ordinary theory can appear only in the term such as $S_e S_\mu$, i.e., in the spin correlation. But there are no such terms in the main order of the quantities (p. 73).