possible mechanisms of discrete spark production is the longitudinal self-focusing channel instability described in [6]. Another mechanism may be the spatial beats between the different frequency components of the laser emission, or spatial beats between the laser emission and the Mandel'shtam-Brillouin scattering components, similar to the beats observed in [7] between the laser emission and the Stokes component of Raman scattering.

The production of the sparks indicates that in some cases the increase of the power in the channel is indeed limited by ionization.

Breakdown was observed in carbon disulfide and nitrobenzene, but not in toluene.

Observation of breakdown in an unfocused beam is reported also in [8], which became known to the authors after this paper was written.

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NONLINEAR OPTICS OF GYROTROPIC MEDIA

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1. The subject of this letter is a discussion of the singularities of propagation of intense light waves in gyrotropic media. Recent success in the construction of powerful sources of visible and ultraviolet radiation make it possible to set up suitable experiments in media having natural optical activity. In nonlinear optics of gyrotropic media one deals with at least two types of problem. On the one hand, effects connected with spatial dispersion appear anew in strong light fields. This set of problems, which we shall call "nonlinear gyrotropy" for short, includes such phenomena as nonlinear rotation of the plane of polarization, nonlinear circular dichroism, etc. Their study can yield new information on the physical properties of the medium. On the other hand, ordinary, linear gyrotropy can change the behavior of already known nonlinear effects. This pertains, in particular, to coherent processes (such as harmonic generation, anti-Stokes Raman scattering, etc.) for which the gyrotropic properties of the medium can change the conditions for optimal interaction. Finally, interest attaches to the study of coherent nonlinear effects under conditions when the "nonlinear gyrotropy" is significant. We develop below a procedure for solving problems in nonlinear optics of gyrotropic media, and discuss effects pertaining to both of the above classes; the calculations were made with optically active media as examples.

2. Equations of nonlinear optics of gyrotropic media. The electromagnetic properties of a medium will be described by material equations in the form

$$D_i = \epsilon_{ij} E_j + \gamma_{ijk} \frac{\partial E_j}{\partial X_k} + \chi_{ijk} E_j E_k + \chi'_{ijkl} E_j \frac{\partial E_k}{\partial X_l} +$$

$$+ \theta_{ijkl} E_j E_k E_l + \Gamma_{ijklm} E_j E_k \frac{\partial E_l}{\partial X_m} . \quad (1)$$

In (1), the tensor γ_{ijk} describes linear gyrotropy (see [1,2], the tensors χ_{ijk} and χ'_{ijkl} effects quadratic in the field ($\chi'/\chi \sim d/\lambda$), and the tensors θ and Γ effects cubic in the field ($\Gamma/\theta \sim d/\lambda$). To describe the wave phenomena in medium (1) it is convenient to use the method of slowly varying amplitudes (see [3]). Representing the interacting waves in the form $\vec{E}_n = \vec{A}_n(\mu\vec{r}, \mu t) \exp i(\omega t - \vec{k}\cdot\vec{r})$, we can obtain approximate equations for the amplitudes $\vec{A}_n$, much simpler than the initial Maxwell equations. We confine ourselves henceforth to the geometric-optics approximation and to unmodulated waves; generalization to include diffraction and temporal monochromaticity is not difficult within the framework of the method of slowly varying amplitudes (see [4]). The modulation of light in gyrotropic media was considered in [5].

3. "Nonlinear gyrotropy." For an isotropic medium or for a cubic crystal, the equation for the complex amplitude $\vec{A}$ takes under the foregoing assumptions the form

$$(\mathbf{k}\vec{\nabla}) \mathbf{A} + \alpha \mathbf{A} + \frac{\omega^2}{2c^2} f_0(\omega) \mathbf{A}\times\mathbf{k} = i \frac{\omega^2}{2c^2} \mathbf{k}^0 \times [\mathbf{K}^0 \times \mathbf{D}^{nl}(\omega)]. \quad (2)$$

Here $f_0(\omega)$ is the gyration constant, $\alpha = (\omega^2/2c^2) I_m \varepsilon(\omega)$, $\vec{D}^{(nl)}(\omega)$ is the Fourier component of the nonlinear induction at the frequency ω, $\vec{k}^0$ is a unit vector along $\vec{k}$. In a monochromatic field the Fourier component $\vec{D}^{(nl)}(\omega)$ can appear only as a result of the last two terms of (1).

The primary cause of the nonlinear gyrotropy is, obviously, the last term of (1); in an isotropic medium it should take the form $\vec{D}^{(nl)} = if_2(\omega)(\vec{A}\cdot\vec{A}^*)[\vec{A}\times\vec{k}]$. Substituting this expression in (2), directing the z axis along the normal to the boundary of the medium, and writing $\vec{A}$ in the form $\vec{A} = \vec{e}_x A_x + \vec{e}_y A_y$, we get in lieu of (2) the system

$$\begin{cases} \dfrac{dA_x}{dz} = -\rho_0 A_y - i\eta_0 A_y - \delta_0 A_x - \rho_2 (\mathbf{A}\mathbf{A}^*) A_y - i\eta_2 (\mathbf{A}\mathbf{A}^*) A_y \\ \dfrac{dA_y}{dz} = \rho_0 A_x + i\eta_0 A_x - \delta_0 A_y + \rho_2 (\mathbf{A}\mathbf{A}^*) A_x + i\eta_2 (\mathbf{A}\mathbf{A}^*) A_x, \end{cases} \quad (3)$$

where $$\rho_{0,2} = \frac{\omega^2}{2c^2 \cos \mathbf{k}\hat{\mathbf{z}}} Re\, f_{0,2}(\omega); \quad \eta_{0,2} = \frac{\omega^2 I_m f_{0,2}(\omega)}{2c^2 \cos \mathbf{k}\hat{\mathbf{z}}};$$

$$\delta_0 = \frac{\alpha}{K \cos \mathbf{k}\hat{\mathbf{z}}} .$$

We see from (3) that the terms with ρ_2 and η_2 lead to the same effects as the terms with ρ_0 and η_0, which describe linear gyrotropy, but the field dependence leads in the former case to a number of singularities. Whereas in a linear medium ($\eta_2 = \rho_2 = 0$) the rotation of a linearly-polarized entering wave ($A_x(0) = A_{x0}$, $A_y(0) = 0$) satisfies the relations $A_x = A_{x0}\cos \rho_0 z$ and $A_y = A_{x0}\sin\rho_{0z}$, and the measure of the dichroism is the ellipticity, defined by the relation

$$\frac{M}{W} = \frac{1}{i} \frac{A_x A_y^* - A_y A_x^*}{|A_x|^2 + |A_y|^2} = th\, 2\eta_0 z \quad , \tag{4}$$

the rotation and the value of M/W in a medium with ρ_2, $\eta_2 \neq 0$ are complicated functions of the coordinate. The rotation is determined by the expression $\rho_0 z + \rho_2 I$, and the damping of the oppositely-circularly-polarized waves by $\delta_0 z + \eta_2 I$ (with $\eta_0 = 0$), where

$$I = \frac{1}{2\eta_2} \ln tg \left[\frac{\pi}{4} + \frac{\eta_2 |A_{x0}|^2}{2\delta_0} (1 - e^{-2\delta_0 z})\right].$$

A situation is possible in which the tensor γ_{ijk} does not lead to rotation but Γ_{iklm} produces rotation (sf. [6]).

Although the term with the tensor θ_{ijkl} is not connected directly with spatial dispersion, allowance for it in a gyrotropic medium can also lead to nonlinear rotation. According to [7], the general form of the corresponding part of $D_i^{(nl)}(\omega)$ is $D_i^{(nl)}(\omega) = aA_i(\vec{A}\cdot\vec{A}^*) + bA_i^*(\vec{A}\cdot\vec{A})$. Substitution of this expression in equations such as (3) shows readily that in a dichroic medium there takes place additional rotation of a linearly polarized entering wave, proportional to $\omega^2/(c^2\cos \hat{kz})\eta_0|A_{x0}|z^2$ (additional rotation of elliptically polarized entering waves occurs also when $\eta_0 = 0$, as first pointed out in [7], and this effect was considered in [10] as applied to waves in a plasma). A definite role can also be played by the nonlinear variation of the wavelength as a result of the term with θ, which leads to a change in the parameter of the spatial dispersion.

4. Spatial dispersion in coherent nonlinear processes. We discuss briefly, in conclusion, some peculiarities in nonlinear wave interactions in the presence of spatial dispersion. We consider second-harmonic generation by a circularly-polarized wave in a crystal of class T-23 (this class includes the recently described crystal $Bi_{12}GeO_{20}$, which has high optic activity, see [8]). If the fundamental-radiation wave propagates along [111] and has right-hand polarization, then the harmonic wave is left-polarized and its amplitude components are:

$$A_x = \beta A_0^2\, e^{i\rho_0(2\omega)z} \frac{[e^{i(\Delta k - \rho_\Sigma)z} - 1]}{\Delta k - \rho_\Sigma}; \quad A_y = iA_x. \tag{5}$$

Here $\Delta k = k(2\omega) = 2k(\omega)$; $\rho_\Sigma = \rho_0(2\omega) + 2\rho_0(\omega)$. Thus, if $\Delta k = \rho_\Sigma$, then the amplitude of the harmonic increases linearly with the distance, jsut as under conditions of exact phase synchronism. The latter means that spatial dispersion can make it possible, in principle, to compensate for the deviations of the phase velocities and greatly increase the efficiency of the nonlinear process. Although in the case of harmonic generation it is difficult to obtain exact compensation, at any rate with the presently available materials (typical values of the ratio

$[k(2\omega) - 2k(\omega)]/\rho_\Sigma$ are $\sim 10 - 15$, and a similar conclusion is reached by the authors of [9], who considered the case of linear polarization of the fundamental radiation), the spatial dispersion can greatly alter the character of the excitation of the anti-Stokes components of stimulated Raman scattering (SRS). Indeed, the spatial dispersion is able in this case tc compensate fully relatively small wave deviations $\Delta k = 2k_\ell - k_S - k_a$ ($\rho_\Sigma = 2\rho(\omega_\ell) + \rho(\omega_S) + \rho(\omega_a)$ for the waves of the appropriate polarizations). A possibility thus arises of effectively exciting anti-Stokes components in optically active media in the laser-beam direction; this can strongly alter the SRS picture and greatly increase the intensity of the anti-Stokes components. Such a compensation can be used in similar fashion in other interactions, too; it is most effective for interactions with small Δk, especially for four-photon parametric amplification.

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POSSIBLE NATURE OF CP-NONINVARIANCE OF WEAK INTERACTIONS

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Wolfenstein proposed in one of his papers [1] that CP-parity violation is due to the fact that the constant for the interaction between the hadron current, $\Delta S = 1$, with the leptons is imaginary. The constants of other types of weak interaction are real.

According to Wolfenstein, CP-noninvariance in nonlepton process would result, as an effect of second order in G, from diagrams of the type shown in Fig. 1, and could be appreciable, since the integral with respect to the relative momentum of the leptons ($p_e - p_\nu$) is not cut off by strong interactions.

The lepton loop relates hadron currents in the form

$$[\delta_{\alpha\beta}(\Lambda^2\phi_1 + k^2\phi_2) + k_\alpha k_\beta \phi_3]\, j_\alpha(\Delta S = \Delta Q = \pm 1)\, j_\beta(\Delta S = 0, \Delta Q = \mp 1),$$

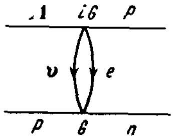

Fig. 1

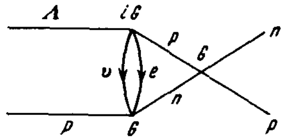

Fig. 2