was determined by the expression obtained from the solution of the second-order secular equation; the Hamiltonian for the interaction between the electrons and the light was taken in the form $H_i = i\hbar e/mc(\nabla\vec{A})$, where $\vec{A}$ is the vector potential of the electromagnetic field, e and m are the charge and mass of the free electron, and c is the speed of light; the relaxation processes were taken into account with the aid of a Lorentz function with a parameter γ characterizing the smearing of the energy levels. For a cubic crystal we obtained

$$\sigma = \frac{1}{\hbar} \cdot \frac{e^2}{\pi^2 \hbar^2} \cdot \sum_g n_g p_g \cdot I,$$

$$I = \frac{\gamma'}{\omega'} \int_0^\infty \frac{dx}{\sqrt{1+x^2}\left[\left(\sqrt{1+x^2} - \omega'\right)^2 + \gamma'^2\right]},$$

$$\gamma' = \gamma/(\hbar\omega_g), \quad \omega' = \omega/\omega_g, \quad \hbar\omega_g = 2|V_g|.$$

Here n_g is the number of physically equivalent Bragg planes g, p_g is the distance from the center of the band to the corresponding Bragg plane in momentum space, and ω is the cyclic frequency of the light. The summation is carried out over the physically nonequivalent Bragg planes.

An analysis of the function $\sigma(\omega)$ shows it to have maxima at the frequencies $\omega_{max} = 2|V_g|/t \approx 2|V_g|$. The coefficient t depends on γ' and its maximum deviation from unity does not exceed 6%. A detailed exposition of this theory will be published separately.

Using the results of this theory and the experimental data for aluminum [3] we get: $|V_{200}| = 0.72 \pm 0.01$ eV, $|V_{111}| = 0.22 \pm 0.03$ eV, in good agreement with data obtained from the de Haas - van Alphen effect [4], namely $V_{200} = 0.76$ eV and $V_{111} = 0.24$ eV.

Comparison of the experimental and theoretical absolute values of $\sigma(\omega)$ shows good agreement between theory and experiment.

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OSCILLATORY SIZE EFFECTS IN METALS WITH ARBITRARY DISPERSION

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It is known that investigations of magnetic oscillatory effects (the de Haas - van Alphen and others) yield valuable information on the structure of the energy spectrum of metals [1]. It is possible to use for this purpose, in principle, oscillatory size effects [2, 3] which have been observed recently in films of bismuth [4, 5] and antimony [6]. In this paper we analyze the information obtained by studying oscillatory size effects in metals at an arbit-

rary quasiparticle dispersion law $\varepsilon(\vec{p})$ (in the quasiclassical case).

The rules for quasiclassical quantization in films, for an arbitrary dispersion law, were derived by I. Lifshitz [7] and by Nedorezov [8] (see a discussion of earlier work on this subject in [8]). They take the form (L is the film thickness):

$$P_z = p_z^1 - p_z^2 = \frac{2\pi}{L}(n+\gamma), \quad n = 1, 2, 3, \ldots, \quad 0 \leqslant \gamma < 1, \tag{1}$$

where the points p_z^1 and p_z^2, shown in the figure, represent two solutions of the equation $\varepsilon(p_x, p_y, p_z^1) = \varepsilon(p_x, p_y, p_z^2) = \varepsilon$ with specified p_x and p_y (the z axis is perpendicular to the surface of the film).

Condition (1) can be written in the equivalent form

$$\Delta_\varepsilon = \frac{2\pi}{T_0}, \quad T_0 = \frac{L}{v_z^1} - \frac{L}{v_z^2}, \quad v_z = \partial\varepsilon/\partial p_z, \tag{2}$$

where Δ_ε is the distance between the quantized levels ($\hbar = 1$) and T_0 is the period of the classical motion of the quasiparticles between the surfaces of the film.

Writing the thermodynamic potential of the metal Ω in the form

$$\Omega = -T\sum_n 2\int \frac{dp_x\,dp_y}{(2\pi)^2} \ln\left[1 + \exp\frac{\mu - \varepsilon_n(p_x, p_y)}{T}\right], \tag{3}$$

where $\varepsilon_n(p_x, p_y)$ are the quasiclassical levels defined by the condition (1), we represent Ω in the form $\Omega_0 = \sum_{s=1}^{\infty}\Omega_s^{osc}$, where separation of the oscillating terms Ω_s^{osc} with the aid of the Poisson formula leads to the formula ($p_0 L \gg 1$):

$$\Omega_s^{osc} = \frac{1}{\pi^2 s^3 L^2}\left(\frac{\partial P_z}{\partial\mu}\right)_{ext}^{-1} \left[\det\left(\frac{\partial^2 P_z}{\partial p_i\,\partial p_k}\right)_{ext}\right]^{-1/2} \sin[sLP_z^{ext}(\mu) - 2\pi s\gamma]\cdot\psi(\lambda), \tag{4}$$

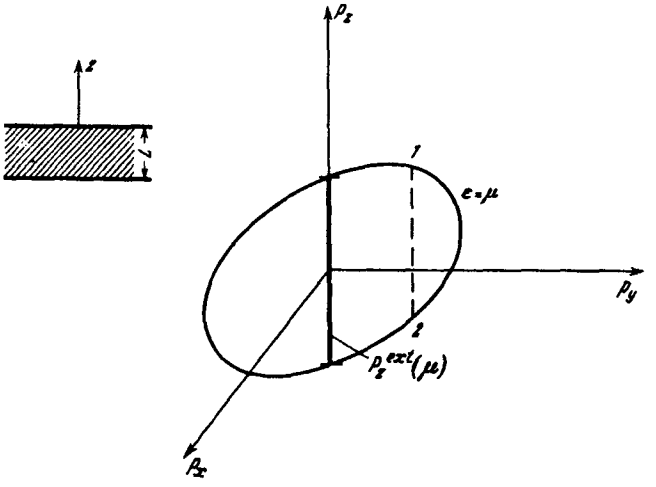
$$\psi(\lambda) = \lambda/\operatorname{sh}\lambda, \quad \lambda = \pi s L T\left(\frac{\partial P_z}{\partial\mu}\right)_{ext} = 2\pi^2 s\, T/(\Delta\varepsilon)_{ext}, \tag{5}$$

where $(\partial P_z/\partial\mu)_{ext}$ is the extremal value, with respect to p_x and p_y, of the derivative $\partial P_z/\partial\varepsilon$ at $\varepsilon = \mu$; p_i and p_k stand for p_x and p_y.

We see from (3) that the thermodynamic potential, and with it any thermodynamic parameter of the metal (for example, the compressibility), oscillates with variation of the film thickness, with a period $\Delta L = 2\pi/P_z^{ext}(\mu)$, where $P_z^{ext}(\mu)$ is the extremal chord of the Fermi surface parallel to the normal to the film surface (see the figure). The temperature dependence of the oscillation amplitude at $T \geq \Delta_\varepsilon$ is of the form $\exp[-2\pi^2 T/(\Delta_\varepsilon)_{ext}]$ where, according to (2), $(\Delta_\varepsilon)_{ext}$ can be represented in the form

$$(\Delta\varepsilon)_{ext} = \frac{2\pi}{L}\left(\frac{1}{|v_z^1|} + \frac{1}{|v_z^2|}\right)_{ext}^{-1}. \tag{6}$$

Although we have considered only oscillations of thermodynamic quantities, it is clear that oscillations of kinetic and other parameters will have the same periods, and their temperature dependence will be determined by the same exponential factors (see [9]).



Thus, a study of oscillatory size effects yields the values of the extremal chords of Fermi surface $P_z^{ext}(\mu)$ and the sums of the reciprocal velocities at the points of intersection of the extremal chord with the Fermi surface. In the case of a centrally-symmetric Fermi surface, it is possible to determine directly the values of the radius vector of the Fermi surface in a given direction n, viz., $p = p(\vec{n})$, and the projection $v(\vec{n}) = \vec{v}\cdot\vec{n}$ of the electron velocity on this direction.*

Although an experimental investigation of the oscillatory size effect is a rather complicated matter and only the first steps have been made in this direction [4-6], it should be noted that in principle this phenomenon affords an opportunity of obtaining most directly information concerning the energy spectrum of the electrons in a metal.

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*The situation is more complicated if the Fermi surface is not convex or not singly-connected. Then the equation $\varepsilon(p_x, p_y, p_z) = \varepsilon$ with specified p_x and p_y can have more than two solutions, and the oscillation picture becomes more complicated. On the other hand, however, this same circumstance yields additional information concerning the spectrum. Thus, if the Fermi surface breaks up into a number of individual sections (e.g., the electron and hole ellipsoids in Bi), then the oscillatory size effect turns out to be sensitive to the relative arrangement of these regions, and thus can yield information on the positions of similar sections of the Fermi surface in the reciprocal-lattice unit cell (we note that a similar possibility was noted earlier in an investigations of the singularities of sound absorption by metals [10]).